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[redacted]

HW8/4.11, 4.24, 4.26, 4.39, 4.73, 4.77, 4.84

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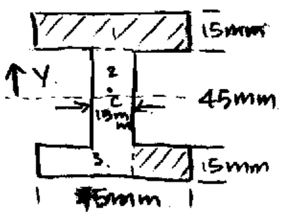
4.11) Knowing that a beam of the cross section shown is bent about a horizontal axis and that the bending moment is 8kN·m. Determine the total force acting on the top flange.



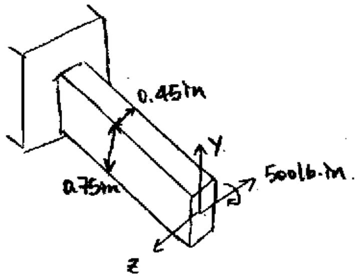
A	Y	YA
$0.001125\,m^2$	0.0675	$7.59\times10^{-5}\,mm^3$
$6.75\times10^{-4}\,m^2$	0.0375	$2.53\times10^{-5}\,mm^3$
$0.001125\,m^2$	0.0075	$8.44\times10^{-6}\,mm^3$
$0.00293\,m^2$		$1.094\times10^{-4}\,mm^3$

$\bar{Y} = 0.0374m$
$= 37.4\,mm$

$$I_x = \tfrac{1}{12}(75mm\times15^3mm) + (75\times15)(30.1)^2 + \tfrac{1}{12}(45^3\times15) + (45\times15)(0.1)^2$$
$$+\tfrac{1}{12}(75mm\times15^3mm) + (75\times15)(29.9)^2$$
$$= 1026855 + 113913 + 1026855$$
$$= 2167623\,mm^4 = 2.17\times10^{-6}\,m^4$$

$$F = \int_0^{0.075}\int_{0.0225}^{0.0375} -\frac{MY}{I} = 0.075\left(-\frac{M}{I}\right)\times\left(\frac{Y^2}{2}\Big|_{0.0225}^{0.0375}\right)$$
$$= 0.075\left(-\frac{8000\,N\cdot m}{2.17\times10^{-6}m^4}\right)\times\left(\frac{0.0375^2-0.0225^2}{2}\right)$$
$$= \underline{-124424\,N} \approx \underline{-124\,kN}$$

4.24) A 500lb·in couple is applied to steel bar. a) Assuming that the couple is applied about the z-axis, determine the max stress and the radius of curvature of the bar. b) solve part a, assuming that the couple is applied about y-axis. Use $E = 29\times10^6\,psi$



a) $\sigma_{max} = \dfrac{MC}{I} = \dfrac{500\,lb{\cdot}in\times(0.75in/2)}{\tfrac{1}{12}(0.45)(0.75)^3}$
$= \dfrac{187.5\,lb{\cdot}in^2}{0.0158\,in^4}$
$= \underline{11851.9\,psi}$

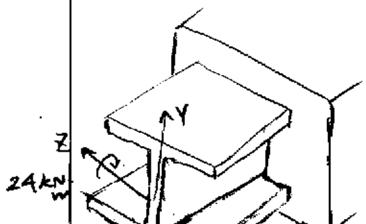
$\rho = \dfrac{EI}{M} = \dfrac{29\times10^6 psi\,(\tfrac{1}{12})(0.45)(0.75)^3}{500} = \underline{917.6\,in}$

b) $\sigma_{max} = \dfrac{MC}{I} = \dfrac{(500)(0.45/2)}{(\tfrac{1}{12})(0.75)(0.45)^3} = \underline{19753.1\,psi}$

$\rho = \dfrac{EI}{M} = \dfrac{29\times10^6 psi\,(\tfrac{1}{12})(0.75)(0.45)^3}{500}$

$= \underline{330.3\,in}$

4.26) A 24 KN·m couple is applied to the W200 ×46.1 rolled-steel beam. a) Assuming that the couple is applied about the z axis, determine the max stress and the radius of curvature of beam. b) Solve part a, assuming the couple is applied about y-axis. Use E = 200 GPa.



$I_{xx} = 45.5 \times 10^6 \, mm^4$ I beam.

$S_x = \frac{I_x}{c} = 448 \times 10^3 \, mm^3$

a) $\sigma_{max} = M/S_x = 24000 \, N\cdot m / (448 \times 10^3 \, mm^3 \times \frac{1 \, m^3}{1000^3 \, mm^3})$

$= 53571429 \, pa = \underline{53.6 \, Mpa}$

$\frac{1}{\rho} = \frac{M}{EI} = \frac{24000 \, N\cdot m}{200 \times 10^9 pa \, (45.5 \times 10^6 mm^4 \times \frac{1 m^3}{1000^4 mm^4})} = 0.0026 \frac{1}{m}$

$\rho = 1/0.0026 \frac{1}{m} = \underline{379.2 m}$

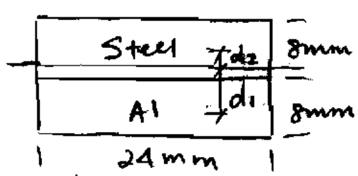
b) $I_y = 15.3 \times 10^6 \, mm^4$ $S_y = 151 \times 10^3 \, mm^3$

$\sigma_{max} = 24000 \, N\cdot m / (151 \times 10^3 mm^3 \times \frac{1 m^3}{1000^3 mm^3}) = 158940397.4 \, pa = \underline{158.9 \, Mpa}$

$\rho = \frac{EI}{M} = \frac{200 \times 10^9 pa \, (15.3 \times 10^6 mm^4) \times (\frac{1 m^3}{1000^4 mm^4})}{24000 \, N\cdot m} = \underline{127.5 m}$

a) 53.6 Mpa, 379 m
b) 158.9 Mpa, 127.5 m

4.39) A steel bar (E_s = 210 GPa) and an Al bar (E_a = 70 GPa) are bonded together to form the composite bar. Determine the max stress in a) Al b) steel, when the bar is bent about a horizontal axis.



$n = \frac{E_2}{E_1} = \frac{210 \, GPa}{70 \, GPa} = 3$ $A = 24 \times 8$

$\bar{y} = \frac{4 \times A + 3 \times 12 \times A}{4 \times A} = 10$

$d_1 = 4 + 2 = 6 mm$
$d_2 = 4 - 2 = 2 mm$

$I_{eff} = I_1 + nI_2$
$= 13312 \, mm^4$

$I_1 = \frac{bh^3}{12} + d_1^2 A = \frac{24 \times 8^3}{12} + 6^2 \times 8 \times 24 = 7936 \, mm^4$

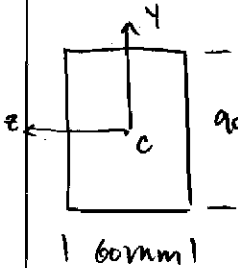
$I_2 = \frac{bh^3}{12} + d_2^2 A = \frac{24 \times 8^3}{12} + 2^2 \times 8 \times 24 = 1792 \, mm^4$

a) $\sigma_{Al} = \frac{-M_z Y}{I_{eff}} = \frac{-(60)(-10) mm}{13,312 mm^4} = \underline{45.07 Mpa}$

b) $\sigma_{st} = \frac{-n M_z Y}{I_{eff}} = \frac{-3 \times 60 N\cdot m \times 6mm}{13,312 mm^4} = \underline{-81.13 Mpa}$

a) 45.1 Mpa
b) -81.1 Mpa

4.73) A beam of cross section shown is made of a steel that is assumed to be elastoplastic $E = 200\,GPa$ & $\sigma_Y = 240\,MPa$. For bending about z-axis, determine the bending moment at which a) yield first occurs. b) the plastic zone at the top & bottom of the bar are 30mm thick.



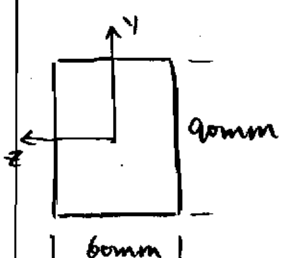
$$M_Y = \frac{2}{3}\sigma_Y b c^2$$
$$= \frac{2}{3}(240 \times 10^6 pa)(0.06)(0.045m)^2$$
$$= \underline{19440\,N \cdot m}$$

$$Y_Y = 45 - 30 = 15\,mm$$

$$M_{ep} = 240 \times 10^6 pa (0.06)(0.045)^2 \left[1 - \frac{1}{3}\left(\frac{15}{45}\right)^2\right]$$
$$= \underline{28080\,N \cdot m}$$

a) 19.44 KN·m
b) 28.1 KN·m

4.77) For the beam indicated, determine a) the fully plastic moment M_p, b) the shape factor of cross section



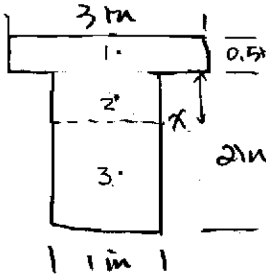
a) $M_{ult} = (240 \times 10^6)(0.06)(0.045)^2$
$= \underline{29160\,N \cdot m}$

b) $K = \frac{M_{ult}}{M_Y} = \frac{29160\,N \cdot m}{19440\,N \cdot m} = \underline{1.5}$

from previous problem

a) 29.2 KN·m
b) 1.500

4.84) Determine the plastic moment, M_p of the cross section shown, assuming the steel to be elastoplastic with a yield strength of 48ksi



$A = (3in)(0.5in) + (2in)(1in)$
$= 3.5 in^2$

$(3in)(0.5in) + (3y) = 1.75$
$y = 0.25 in$

$R_1 = A_1 \sigma_Y = 3in(0.5in) \times 48000 psi = 72000$
$R_2 = A_2 \sigma_Y = (1in)(0.25) \times 48000 psi = 12000$
$R_3 = A_3 \sigma_Y = (1in)(2-0.25) \times 48000 psi = 84000$

$M_p = (0.5 in)(R_1) + (0.125in)(R_2) + (0.875in)(R_3)$
$= (0.5)(72000) + (0.125)(12000) + (0.875)(84000)$
$= 111000 lb \cdot in$
$= \underline{111 kips \cdot in}$

111.0 kip·in