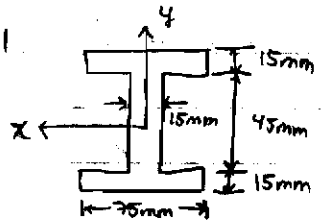
Michael Perez
10/21/06
Civil Engineering 130
[redacted]

70

HOMEWORK #8

PROBLEMS # 4.11, 4.24, 4.26, 4.39, 4.73, 4.77, 4.84

4.11

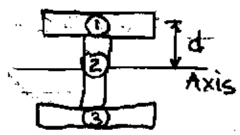


$\sigma_x = -\frac{My}{I}$

$F = -\frac{M\bar{y}A}{I}$

$M = 8\,kN\cdot m$

$\bar{y}$: Centroidal coordinate from centroid of whole area to centroid of partial area.



$I_1 = \frac{1}{12}bh_1^3 + A_1d_1^2$

$= \frac{1}{12}(75mm)(15mm)^3 + (75mm)(15mm)(30mm)^2$

$= 21093.75\,mm^4 + 1012500\,mm^4 \Rightarrow I_1 = 1.034\times10^6\,mm^4$

$d = 7.5mm + 22.5mm = 30\,mm$

$I_2 = \frac{1}{12}b_2h_2^3 + A_2d_2^2{}^{\,0} = \frac{1}{12}b_2h_2^3$

$= \frac{1}{12}(15mm)(45mm)^3 \Rightarrow I_2 = 1.139\times10^5\,mm^4$

$\bar{y} = 30mm = 0.03m$

$I_3 = I_1 \Rightarrow I_3 = 1.034\times10^6\,mm^4$

$I = I_1 + I_2 + I_3$

$= 2(1.034\times10^6 mm^4) + 1.139\times10^5 mm^4 \Rightarrow I = 2.1811\times10^6\,mm^4$

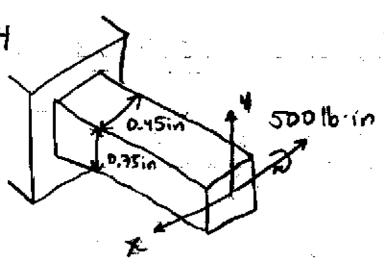
$2.1811\times10^6 mm^4\left(\frac{1cm^4}{(10\,mm)^4}\right)\left(\frac{1m^4}{(100cm)^4}\right) = 2.1811\times10^{-6}\,m$

$A_1 = (75mm)(15mm) \Rightarrow A = 1125mm \Rightarrow A = 1.125\times10^{-3}\,m^2$

$F = -\frac{(8\,kN\cdot m)(0.030m)(1.125\times10^{-3}m^2)}{(2.1811\times10^{-6}m^4)} \Rightarrow -\frac{2.7\times10^{-4}\,kN\cdot m^4}{2.1811\times10^{-6}}$

$\Rightarrow \boxed{F = -123.791\,kN}$

4.24



$M = 500\,lb\cdot in \quad E = 29\times10^6\,psi$

(a) σ_m = maximum stress

$I = \frac{1}{12}bh^3 = \frac{1}{12}(0.45in)(0.75in)^3$

$I = 0.0158\,in^4$

$c = \frac{1}{2}h = \frac{1}{2}(0.75in) \Rightarrow c = 0.375in$

$\sigma_m = \frac{Mc}{I} = \frac{(500\,lb\cdot in)(0.375in)}{0.0158in^4} \Rightarrow \sigma_m = 11851.852\,psi$

$\Rightarrow \boxed{\sigma_m = 11.852\,ksi}$

$\frac{1}{\rho} = \frac{M}{EI} \Rightarrow \rho = \frac{EI}{M}$

$\rho = \frac{(29\times10^6 psi)(0.0158\,in^4)}{500\,lb\cdot in} \Rightarrow \rho = 917.578in \Rightarrow \boxed{\rho = 76.465\,ft}$

(b.) $I = \frac{1}{12}bh^3 = \frac{1}{12}(0.75\text{in})(0.45\text{in})^3 \Rightarrow I = 0.005695\ \text{in}^4$

$c = \frac{1}{2}h \Rightarrow c = \frac{1}{2}(0.45\text{in}) \Rightarrow c = 0.225\text{in}$

$\sigma_m = \frac{Mc}{I} = \frac{(500\ \text{lb-in})(0.225\text{in})}{5.695\times10^{-3}\text{in}^4} \Rightarrow \sigma_m = 19753.086\ \text{psi}$

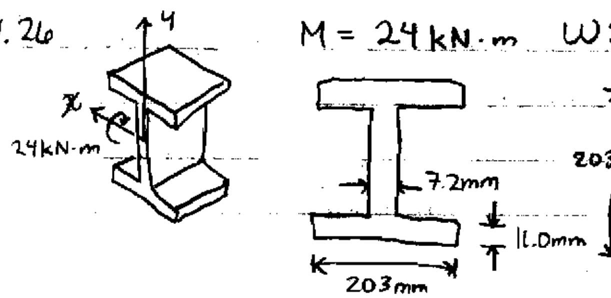
$\boxed{\sigma_m = 19.753\ \text{ksi}}$ ✓

$\frac{1}{\rho} = \frac{M}{EI} \Rightarrow \rho = \frac{EI}{M}$

$\rho = \frac{(29\times10^6\text{psi})(5.695\times10^{-3}\text{in}^4)}{(500\ \text{lb}\cdot\text{in})} \Rightarrow \rho = 330.328\ \text{in} \Rightarrow \boxed{\rho = 27.527\ \text{ft}}$ ✓

4.26

$M = 24\ \text{kN}\cdot\text{m}$ W200×46.1 $E = 200\ \text{GPa}$



From Appendix C:

$I_z = 45.5\times10^6\ \text{mm}^4$

$S_z = 448\times10^3\ \text{mm}^3$

$I_y = 15.3\times10^6\ \text{mm}^4$

$S_y = 151\times10^3\ \text{mm}^3$

$I_z = 45.5\times10^{-6}\ \text{m}^4$ $I_y = 15.3\times10^{-6}\ \text{m}^4$

$S_z = 448\times10^{-6}\ \text{m}^3$ $S_y = 151\times10^{-6}\ \text{m}^3$

(a.) $\sigma_z = \frac{M}{S_z} = \frac{24\ \text{kN}\cdot\text{m}}{448\times10^{-6}\ \text{m}^3} \Rightarrow \sigma_z = 5.357\times10^4\ \text{kPA} \Rightarrow \boxed{\sigma_z = 53.571\ \text{MPa}}$ ✓

$\frac{1}{\rho_z} = \frac{M}{EI_z} \Rightarrow \rho_z = \frac{EI_z}{M}$ $\rho_z = \frac{(200\times10^6\ \text{kPa})(45.5\times10^{-6}\text{m}^4)}{(24\ \text{kN-m})}$

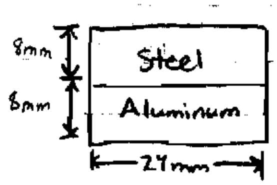
$\boxed{\rho_z = 379.167\ \text{m}}$ ✓

(b.) $\sigma_y = \frac{M}{S_y} = \frac{24\ \text{kN-m}}{151\times10^{-6}\text{m}^3} \Rightarrow \sigma_y = 1.589\times10^5\ \text{kPA} \Rightarrow \boxed{\sigma_y = 158.940\ \text{MPa}}$ ✓

$\frac{1}{\rho_y} = \frac{M}{EI_y} \Rightarrow \rho_y = \frac{EI_y}{M}$ $\rho_y = \frac{(200\times10^6\ \text{kPa})(15.3\times10^{-6}\text{m}^4)}{(24\ \text{kN-m})}$

$\boxed{\rho_y = 127.5\ \text{m}}$ ✓

4.39

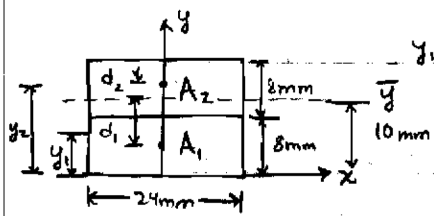


$E_s = 210\ \text{GPa} = 210\times10^9\ \text{Pa}$

$E_A = 70\ \text{GPa} = 70\times10^9\ \text{Pa}$

$M = 60\ \text{N-m}$

$n_1 = 1.0$ $n_2 = \frac{E_s}{E_A} = \frac{E_2}{E_1} = \frac{210\times10^9\ \text{Pa}}{70\times10^9\ \text{Pa}} \Rightarrow n_2 = 3$



$y_1 = 4mm \quad y_2 = 12mm \quad A_1 = A_2 = A = (8mm)(24mm)$

$A = 192mm^2$

$$\bar{y} = \frac{y_1 A_1 + n y_2 A_2}{A_1 + nA_2}$$

$$\bar{y} = \frac{(4mm)(192mm^2) + 3(12mm)(192mm^2)}{192mm^2 + 3(192mm^2)}$$

Neutral Axis is 10mm above bottom

$$\bar{y} = \frac{768mm^3 + 6912mm^3}{768mm^2} \Rightarrow \bar{y} = 10mm$$

$$I_A = I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2$$
$$= \frac{1}{12}(24mm)(8mm)^3 + (192mm^2)(6mm)^2$$
$$= 1024mm^4 + 6912mm^4 \Rightarrow I_1 = 7936mm^4$$

$$I_s = I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2$$
$$= \frac{1}{12}(24mm)(8mm)^3 + (192mm^2)(2mm)^2$$
$$= 1024mm^4 + 768mm^4 \Rightarrow I_2 = 1792mm^4$$

$$I_{eff} = I_1 + nI_2$$
$$= 7936mm^4 + 3(1792mm^4)$$
$$I_{eff} = 13312mm^4$$
$$= 1.331 \times 10^{-8} m^4$$

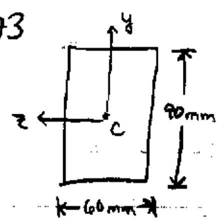
(a) Aluminum:

$$\sigma_A = -\frac{My}{I_{eff}} = -\frac{(60 N\cdot m)(-0.01m)}{1.331 \times 10^{-8} m^4} \Rightarrow \sigma_A = 45.072 \times 10^6 Pa \Rightarrow \boxed{\sigma_A = 45.072 MPa}$$

(b.) Steel: $y = 16mm - 10mm = 6mm \Rightarrow y = 0.006m$

$$\sigma_s = -\frac{nMy}{I_{eff}} = \frac{3(60 N\cdot m)(0.006m)}{-1.331 \times 10^{-8} m^4} \Rightarrow \sigma_s = 81.1298 \times 10^6 Pa \Rightarrow \boxed{\sigma_s = 81.1298 MPa}$$

4.73



$E = 200GPa = 200 \times 10^9 Pa$

$\sigma_y = 240 MPa = 240 \times 10^6 Pa$

(a.) $I = \frac{1}{12} bh^3 = \frac{1}{12}(60mm)(90mm)^3$

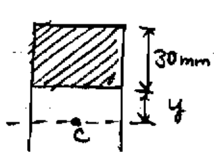
$$= 364.5 \times 10^4 mm^4 \Rightarrow I = 3.645 \times 10^{-6} m^4$$

$$c = \frac{1}{2}h = \frac{1}{2}(90mm) \Rightarrow c = 45mm \Rightarrow c = 0.045m$$

$$M_y = \frac{\sigma_y I}{c} = \frac{(240 \times 10^6 Pa)(3.645 \times 10^{-6} m^4)}{0.045m} \Rightarrow M_y = 19440 N\cdot m$$

$$\Rightarrow \boxed{M_y = 19.44 kN\cdot m}$$

(b.)



$c = \frac{1}{2}h = \frac{1}{2}(90mm) \Rightarrow c = 45mm$

$y = 45mm - 30mm = 15mm$

$$M_{ep} = \sigma_y bc^2 \left[1 - \frac{1}{3}\left(\frac{y}{c}\right)^2\right]$$

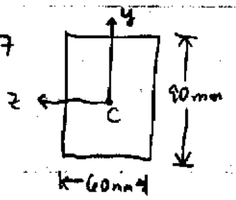
$M_{ep} = (240\times10^6 Pa)(0.06m)(0.045m)^2 \left[1 - \frac{1}{3}\left(\frac{0.015m}{0.045m}\right)^2\right]$

$= (29160 N\cdot m)(1 - 0.037)$

$= (29160 N\cdot m)(0.963) \Rightarrow M_{ep} = 28080 N\cdot m$

$\Rightarrow \boxed{M_{ep} = 28.08 kN\cdot m}$ ✓

4.77



$E = 200GPa = 200\times10^9 Pa$

$\sigma_y = 240 MPa = 240\times10^6 Pa$

(a) $M_p = \sigma_y Z$

$= \sigma_y bc^2 = (240\times10^6 Pa)(0.06m)(0.045m)^2$

$\Rightarrow M_p = 29160 N\cdot m$

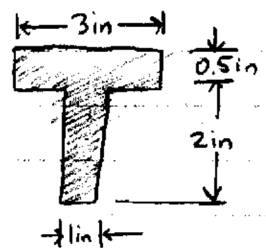
$\therefore \boxed{M_p = 29.16 kN\cdot m}$ ✓

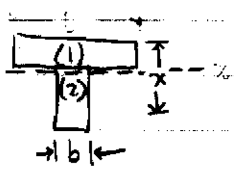
(b.) $k = \frac{M_p}{M_y} = \frac{29.16 kN\cdot m}{19.44 kN\cdot m} \Rightarrow \boxed{k = 1.5}$ ✓

$M_y = 19.44 kN\cdot m$ (from 4.73)

$M_p = 29.16 kN\cdot m$ (from part (a))

4.84





$A = (3in)(0.5in) + (1in)(2in)$ $\sigma_y = 48 ksi$

$= 1.5 in^2 + 2 in^2 \Rightarrow A = 3.5 in^2$

$\frac{1}{2}A = 1.75 in^2$

$x = \frac{\frac{1}{2}A}{b} = \frac{1.75 in^2}{1 in} \Rightarrow x = 1.75 in$

Neutral Axis Lies 1.75 in above bottom

$\bar{y}_1 = \frac{1}{2}(0.75 in) \Rightarrow \bar{y}_1 = 0.375 in$

$\bar{y}_2 = \frac{1}{2}(1.75 in) \Rightarrow \bar{y}_2 = 0.875 in$

$M_p = \sigma_y(A_1\bar{y}_1 + A_2\bar{y}_2)$

$= (48 ksi)[(1.5 in^2)(0.375 in) + (2 in^2)(0.875 in)]$

$= (48 ksi)[0.5625 in^3 + 1.75 in^3]$

$\Rightarrow \boxed{M_p = 111 \text{ kips-in}}$ ✓