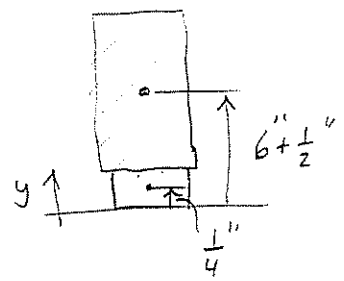
HW #9 : 4.42, 89, 102, 121, 5.50, 5.57, 98, 101

4.42



$$y_{NA} = \frac{\Sigma EAy}{\Sigma EA} = \frac{1.8\times12\times6\times6.5 + 29\times5\times\frac{1}{2}\times\frac{1}{4}}{1.8\times12\times6 + 29\times5\times\frac{1}{2}} = 4.258''$$

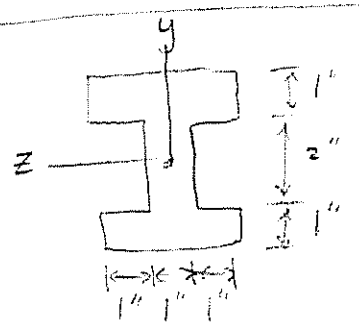
$$I_w = \frac{6\cdot12^3}{12} + 6\cdot12\cdot(6.5-y_{NA})^2 + \frac{E_s}{E_w}\left(5\cdot\frac{1}{2}^3/12 + 5\cdot\frac{1}{2}\cdot\left(\frac{1}{4}-y_{NA}\right)^2\right)$$

$$I_w = 1873.8\ in^4$$

$$\sigma_{w,max} = -\frac{M(12.5-y_{NA})}{I_w} = \frac{-(450\ k\text{-}in)(12.5-y_{NA})}{1873.8\ in^4} = \underline{-1.98\ ksi}$$

$$\sigma_{s,max} = -\frac{E_s}{E_w}\cdot\frac{M(-y_{NA})}{I_w} = -\frac{29}{1.8}\cdot\frac{450\times(-4.258)}{1873.8} = \underline{16.5\ ksi}$$

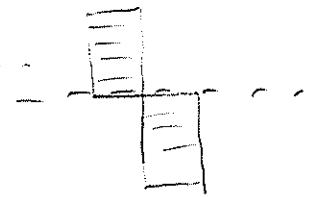
4.89



$$M_p = 2\times\sigma_Y\left(1\times3\times1.5 + 1\times1\times\frac{1}{2}\right) = (10\ in^3)\,\sigma_Y$$

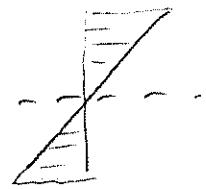
$$= \sigma_Y \Sigma A_i y_i = \int_A \sigma_Y\cdot y\,dA$$

$\sigma_Y = 42\ ksi$ $M_p = 420\ k\text{-}in$



UNLOADING : $I = \frac{3\cdot4^3 - 2\cdot2^3}{12} = 14.67\ in^4$

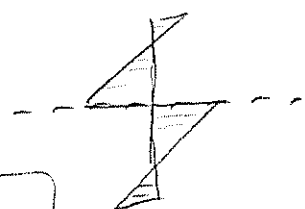
$$\Delta M = -M_p \qquad \Delta\sigma = \frac{M_p\cdot y}{I}$$



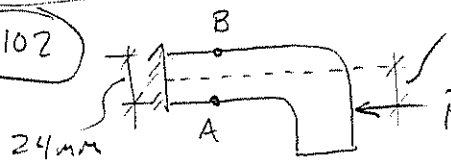
RESIDUAL : $y=0$ to $2''$ $\sigma_{res} = -\sigma_y + \frac{M_p\cdot y}{I} = -\sigma_Y + \frac{10\cdot\sigma_Y\cdot y}{I}$

$$\sigma_{res}(y=1) = \sigma_y\left(-1 + \frac{10.1}{I}\right) = \underline{-13.4\ ksi}$$

$$\sigma_{res}(y=2) = \sigma_y\left(-1 + \frac{10.2}{I}\right) = \underline{15.3\ ksi}$$

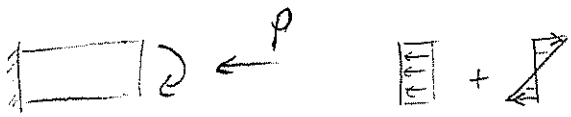


4.102



$$\frac{P}{A} = \frac{8000\ N}{24\times30\ mm} = 11.1\ MPa$$

$$\frac{(P\times33)\cdot12}{I} = 91.7\ MPa$$

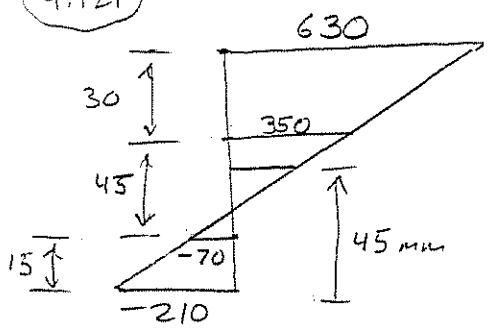


$A = 24\cdot30$ $I = 30\cdot24^3/12$

B : $\sigma = -11.1 + 91.7 = \underline{80.56\ MPa}$

A : $\sigma = -11.1 - 91.7 = \underline{-102.8\ MPa}$

4.121



$$\frac{350-(-70)}{45\,mm} = -K = 9\tfrac{1}{3}\ \mu/mm$$

$$30+45+15 = 90\ mm$$

$$y_{NA} = 90/2 = 45\ mm$$

$$\varepsilon_{NA} = -210 + 9\tfrac{1}{3}\cdot 45 = +210\ \mu$$

$$P = E\cdot\varepsilon_{NA}\cdot A = (200\ GPa)(210\ \mu)(25\times 90) = \underline{\underline{94.5\ kN}}$$

$$M = EI\cdot K = (200\ GP)(25\cdot 90^3/12)(-9\tfrac{1}{3}\times 10^{-6}) = -2835\ kN\cdot mm$$

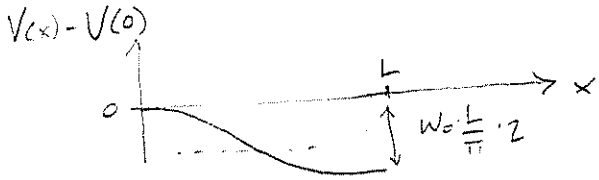
$$M = -P\cdot d = -94.5\cdot d = -2835,\quad \underline{\underline{d = 30\ mm}}$$

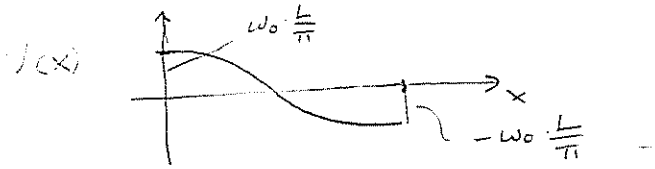
5.50

$$V(x)-V(0) = -\int_0^x w(x)\,dx = -\int_0^x w_0 \sin\frac{\pi\cdot x}{L}\,dx = +w_0\cdot\frac{L}{\pi}\cdot\cos\frac{\pi x}{L}\Big|_0^x$$

$$V(x)-V(0) = w_0\cdot\frac{L}{\pi}\left(\cos\frac{\pi\cdot x}{L}-1\right)$$

$$V(0) = \frac{\int_0^L w(x)\cdot dx}{2} = \frac{-w_0\cdot\frac{L}{\pi}\cdot\cos\frac{\pi x}{L}\Big|_0^L}{2} = -\frac{w_0\cdot\frac{L}{\pi}\left(-1-1\right)}{2} = +w_0\cdot\frac{L}{\pi}\cdot\frac{2}{2} = \frac{w_0 L}{\pi}$$



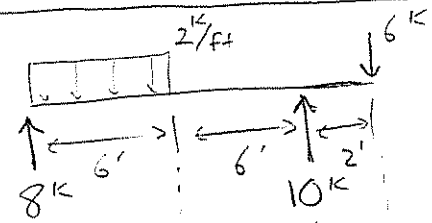


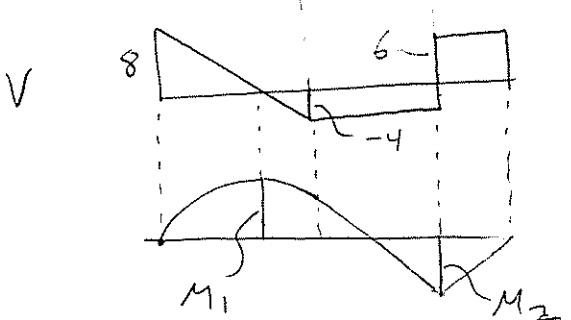
$$V(x) = w_0\cdot\frac{L}{\pi}\cos\frac{\pi\cdot x}{L}$$

$$M(x) - \cancel{M(0)}^{\,0} = \int_0^x V(x)\,dx = w_0\cdot\frac{L}{\pi}\cdot\frac{L}{\pi}\cdot\sin\frac{\pi x}{L}\Big|_0^x = w_0\frac{L^2}{\pi^2}\sin\frac{\pi x}{L}$$

$$M(x) = \frac{w_0 L^2}{\pi^2}\sin\frac{\pi x}{L}$$

5.57





$$\sigma_{max} = -\frac{M_{max}\cdot(\pm c)}{I_x} = \pm\frac{M_{max}}{S}$$

$$W6\times 31:\ S_x = 27.5\ in^3$$

$$\sigma_{max} = \pm\frac{16\times 12}{27.5} = \underline{\underline{7\ ksi}}$$

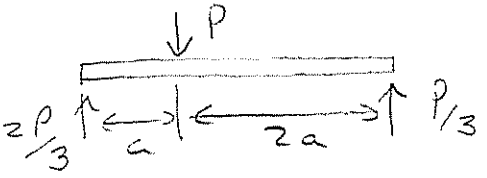
$$M_1 = \tfrac{1}{2}\cdot 8^K\left(\tfrac{2}{3}\cdot 6^{ft}\right) = 16^{k\text{-}ft} \Leftarrow M_{max}$$

$$M_2 = -6^K\cdot 2' = -12^{k\text{-}ft}$$

5.98

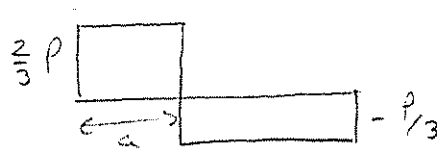
$w(x) = 0$

$V(x) = -P\langle x-a\rangle^0 + \cancel{V(0)}^{\frac{2P}{3}}$



$M(x) = -P\langle x-a\rangle + \frac{2P}{3}\cdot x + \cancel{M(0)}^{0}$

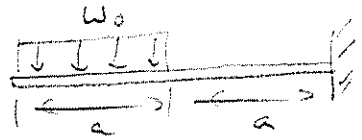
$M(2a) = -P\langle a\rangle + \frac{2P}{3}\cdot 2a = -P\cdot a + \frac{4}{3}P\cdot a = \frac{1}{3}Pa$



✓

5.101

$w(x) = w_0\left(1 - \langle x-a\rangle^0\right)$



$V(x) - \cancel{V(0)}^{0} = -\int_0^x w(x)\,dx = -w_0\left(x - \langle x-a\rangle^1\right)$

$M(x) = -w_0\left(\frac{1}{2}x^2 - \frac{1}{2}\langle x-a\rangle^2\right)$

$M(2a) = -w_0\left(\frac{1}{2}\cdot 4a^2 - \frac{1}{2}\langle a\rangle^2\right)$

$= -w_0\left(2a^2 - \frac{1}{2}a^2\right) = -1.5\,w_0 a^2$