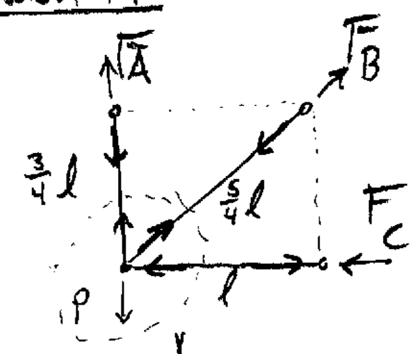
Discussion 14

(119)



$$F_A + \frac{\frac{3}{4}}{\sqrt{\frac{9}{16}+1}} F_B = P, \quad F_A + \frac{3}{5} F_B = P \quad F_B = \frac{5}{3}(P - F_A)$$

$$\frac{\partial F_B}{\partial F_A} = -\frac{5}{3}$$

$$\frac{3/4}{\sqrt{25/16}} = \frac{3/4}{5/4} = \frac{3}{5}$$

$$F_C = -\frac{4}{5} \cdot F_B = -\frac{4}{3}(P - F_A), \quad \frac{\partial F_C}{\partial F_A} = \frac{4}{3}$$

$$\Delta_A = 0 = \sum \frac{P_i l_i}{EA_i} \cdot \frac{\partial P_i}{\partial F_A}$$

$$0 = \frac{F_A \cdot \frac{3}{4} l}{EA} + \frac{\frac{5}{3}(P - F_A) \cdot \frac{5}{4} l}{EA} \cdot -\frac{5}{3} + \frac{-\frac{4}{3}(P - F_A) \cdot l}{EA} \cdot \frac{4}{3}$$

$$0 = F_A \cdot \frac{3}{4} + \frac{5}{3}(P - F_A) \cdot \frac{5}{4} \cdot -\frac{5}{3} + -\frac{4}{3}(P - F_A) \cdot \frac{4}{3}$$

$$0 = \frac{3}{4} F_A - \frac{125}{36} P + \frac{125}{36} F_A - \frac{16}{9} P + \frac{16}{9} \cdot F_A$$

$$0 = \left(\frac{27}{36} + \frac{125}{36} + \frac{64}{36}\right) F_A - \left(\frac{125}{36} + \frac{64}{36}\right) P$$

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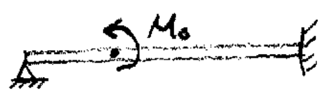
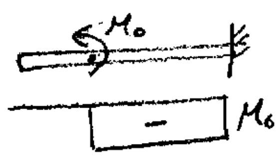
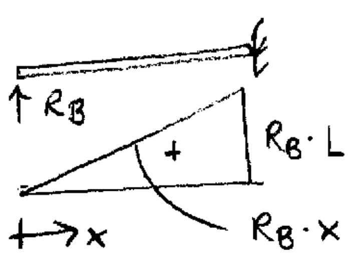






$x = 0$ to a: $M(x) = R_B \cdot x$, $\frac{\partial M}{\partial R_B} = x$

$x = a$ to L: $M(x) = R_B \cdot x - M_0$, $\frac{\partial M}{\partial R_B} = x$

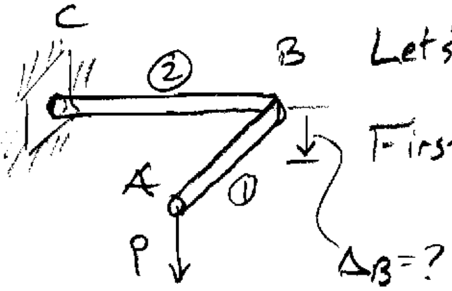
$$\Delta_B = 0 = \int_0^L \frac{M(x)}{EI} \cdot \frac{\partial M(x)}{\partial R_B} dx = \int_0^a \frac{R_B \cdot x \cdot x}{EI} dx + \int_a^L \frac{(R_B \cdot x - M_0) \cdot x}{EI} dx$$

$$0 = \frac{R_B}{EI} \cdot \frac{1}{3} x^3 \Big|_0^a + \frac{R_B}{EI} \cdot \frac{1}{3} x^3 \Big|_a^L - \frac{M_0}{EI} \cdot \frac{1}{2} x^2 \Big|_a^L$$

$$0 = R_B \cdot \frac{1}{3} a^3 + R_B \cdot \frac{1}{3} L^3 - R_B \cdot \frac{1}{3} a^3 - M_0 \cdot \frac{1}{2} L^2 + M_0 \cdot \frac{1}{2} a^2$$

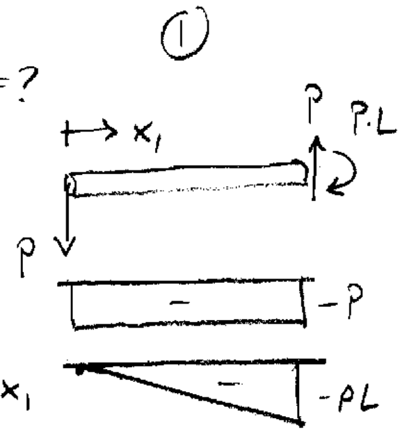
$$0 = R_B \cdot \frac{1}{3} L^3 - M_0 \cdot \frac{1}{2}(L^2 - a^2)$$

$$R_B = \frac{3}{2} \cdot \frac{(L^2 - a^2)}{L^3} M_0$$



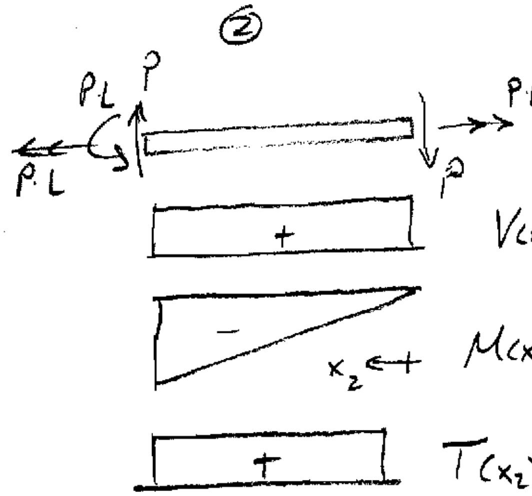
Let's find vert. defl. @ B

First, find internal forces in each member.



$V(x_1) = -P$

$M(x_1) = -P \cdot x_1$



$V(x_2) = P$

$M(x_2) = -P \cdot x_2$

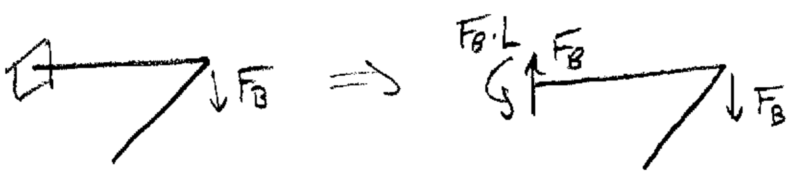
$T(x_2) = PL$

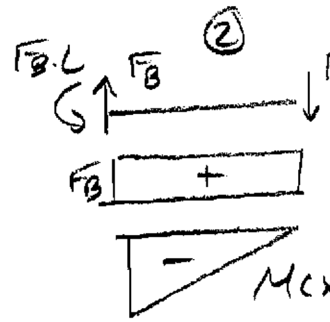
Need force at B to find Δ_B

$$\Delta_B = \frac{\partial U}{\partial F_B} = \int \frac{M(x)}{EI} \cdot \frac{\partial M(x)}{\partial F_B}\, dx + \alpha \int \frac{V(x)}{GA} \cdot \frac{\partial V(x)}{\partial F_B}\, dx + \text{Torsion} + \ldots$$

Need the $\frac{\partial M(x)}{\partial F_B}$, $\frac{\partial V(x)}{\partial F_B}$, $\frac{\partial T(x)}{\partial F_B}$, etc. terms for each bar.

Apply dummy load F_B





$V(x_2) = F_B$

$M(x_2) = -F_B \cdot x$

F_B only causes internal forces in member ②

$$\frac{\partial M(x_1)}{\partial F_B} = 0 \quad \frac{\partial M(x_2)}{\partial F_B} = -x_2 \quad \frac{\partial V(x_1)}{\partial F_B} = 0 \quad \frac{\partial V(x_2)}{\partial F_B} = 1$$

$$\frac{\partial T(x_1)}{\partial F_B} = \frac{\partial T(x_2)}{\partial F_B} = 0$$

$$\Delta_B = \int_0^L \frac{M(x_2)}{EI} \cdot \frac{\partial M(x_2)}{\partial F_B}\, dx_2 + \alpha \int_0^L \frac{V(x_2)}{GA} \cdot \frac{\partial V(x_2)}{\partial F_B}\, dx_2$$ ⟸ only non-zero terms of $\partial U / \partial F_B$

$$\Delta_B = \int_0^L \frac{-P \cdot x_2}{EI} \cdot (-x_2)\, dx + \frac{10}{9} \int_0^L \frac{P}{GA} \cdot 1 \cdot dx_2$$

$$\Delta_B = + \frac{P}{EI} \cdot \frac{1}{3} L^3 + \frac{10}{9} \frac{P}{GA} \cdot L$$ (downwards, since F_B is directed downwards and answer came out positive)