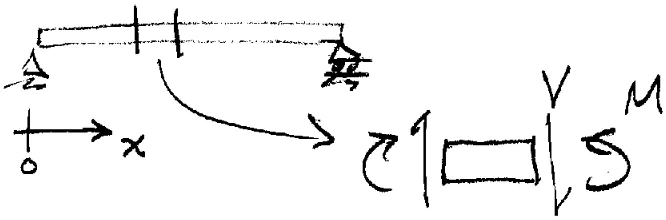
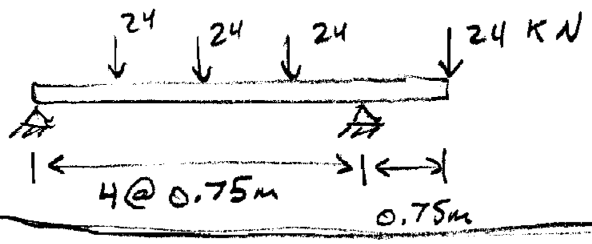
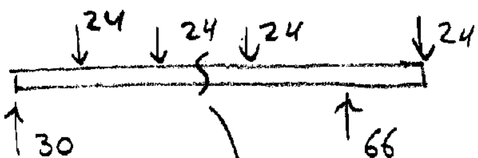
SIGN CONVENTION:



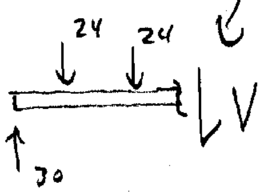
Thus, shear is $\underbrace{\text{sum of all forces}}_{\text{vertical force, positive up}}$ as you travel along beam.

Prob 5.9



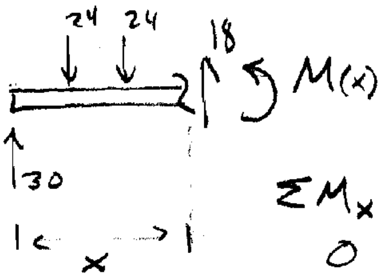


solve for reaction forces using statics.



$V = 30 - 2 \times 24 = -18 \text{ kN}$

THIS IS THE CONCEPT OF INTERNAL SHEAR



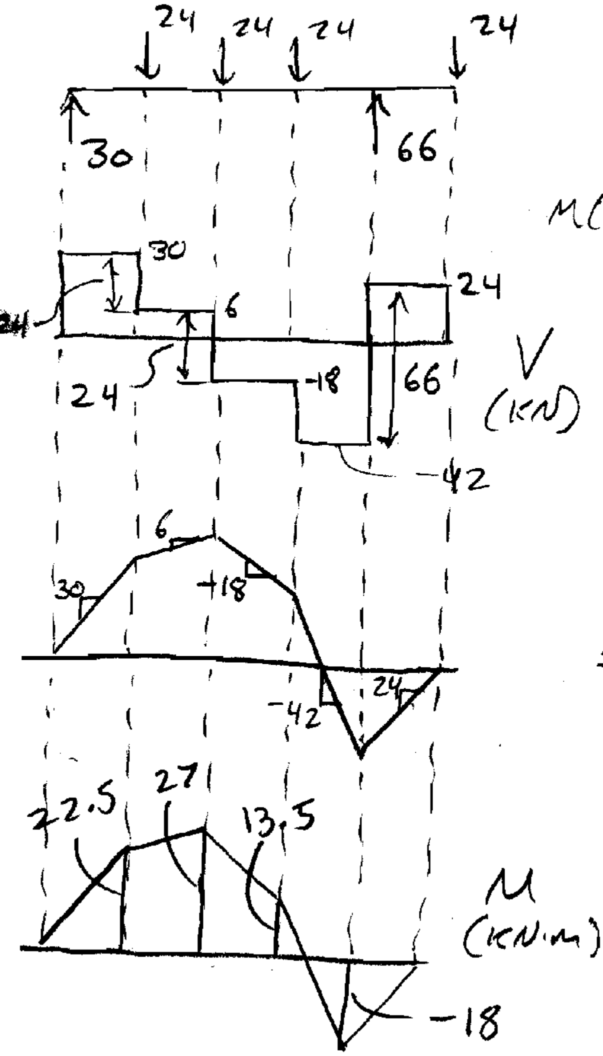
THIS IS THE INTERNAL MOMENT

$\Sigma M_x = 0 \quad \circlearrowright +$

$0 = 30 \cdot x - 24(x - 0.75) - 24(x - 2(.75)) - \underline{M(x)}$

solve for M(x)

INSTEAD OF SOLVING FOR ONE POINT, DRAW A DIAGRAM SHOWING ALL POINTS.



<u>x: 0 to 0.75</u>

$V = 30$

$M(.75) - M(0) = \int_0^{0.75} 30\,dx = \underline{30 \cdot 0.75} = 22.5$

area under shear diagram

<u>x: 0.75 to 1.5</u>

$V = 6$

$M(1.5) - M(.75) = \int_{0.75}^{1.5} 6\,dx = \underline{6 \cdot 0.75} = 4.5$

area under shear diagram

<u>x: 1.5 to 2.25</u>

$V = -18$

$M(2.25) - M(1.5) = \underline{-18(.75)} = -13.5$

area under shear diagram

<u>x: 2.25 to 3</u>

$V = -42$

$\Delta M = -42(.75) = -21.5$

<u>x: 3 to 3.75</u>

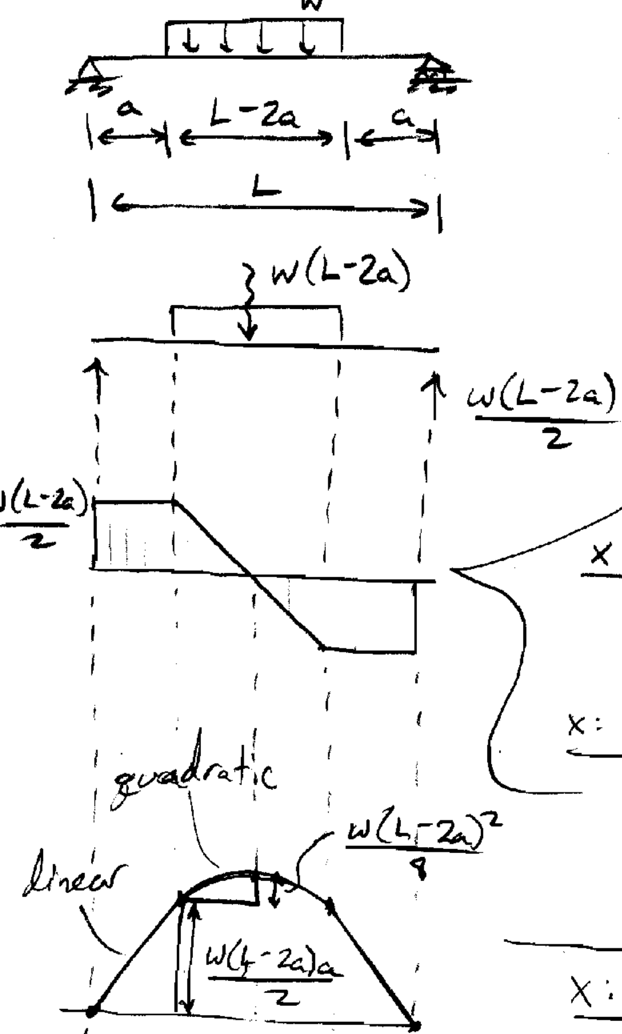
$V = 24$

$\Delta M = 24(.75) = 18$

[redacted]

GRAPHICAL APPROACH (5.5)

$$\frac{dV}{dx} = w(x) \qquad \frac{dM}{dx} = V(x)$$



MOMENT IS ZERO AT SIMPLY SUPPORTED END

x : L-a to L : $w(x) = 0$

$$\Delta V = \int_{L-a}^{L} 0 \, dx = 0$$

x : 0 to a $\quad w(x) = 0$

$$\Delta V = \int_0^a 0 \, dx = 0$$

x : a to L-a $\quad w(x) = -w \rightarrow$ constant

$$\Delta V = \int_a^{L-a} -w \, dx = -w(L-2a)$$

↳ linear

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x : 0 to a : $\Delta M = \int_0^a V(x)\,dx = \frac{w(L-2a)}{2} \cdot a$

↳ linear ↳ constant

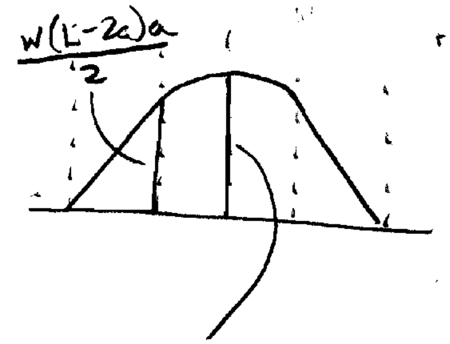
↳ slope $= V(x) = \frac{w(L-2a)}{2}$

x : a to L/2 : $\Delta M = \int_a^{L/2} V(x)\,dx = \frac{1}{2} \frac{w(L-2a)}{2} \cdot \frac{L-2a}{2}$

↳ quadratic ↳ linear

initial slope : $V(a) = \frac{w(L-2a)}{2}$

final slop : $V(L/2) = 0$



$$\frac{w(L-2a)a}{2} + \frac{w(L-2a)^2}{8}$$

x : L/2 to L-a : $\Delta M = \int_{L/2}^{L-a} V(x)\,dx = -\frac{1}{2} \frac{w(L-2a)}{2} \cdot \frac{L-2a}{2}$

initial slope : $V(L/2) = 0$

final slope : $V(L-a) = -\frac{w(L-2a)}{2}$

x : L-a to L : $\Delta M = \int_{L-a}^{L} V(x)\,dx = -\frac{w(L-2a)}{2} \cdot a$

↳ linear ↳ constant

↳ slope $= V(x) = -\frac{w(L-2a)}{2}$

FORMULA APPROACH

$V(0) = w(L-2a)/2$

5.5

$\frac{dV}{dx} = w(x) \quad \frac{dM}{dx} = V(x)$

x: 0 to a

$w(x) = 0$

$V(x) - V(0) = \int_0^x 0\,dx = 0, \quad V(x) = V(0) = w(L-2a)/2$

$M(x) - \cancel{M(0)}^{\to 0} = \int_0^x \frac{w(L-2a)}{2}dx = \frac{w(L-2a)}{2}\Big|_0^x = \frac{w(L-2a)x}{2}$

CHECK: $M(a) = \frac{w(L-2a)\cdot a}{2}$ OK!

x: a to L−a

$w(x) = -w$

$V(x) - V(a) = \int_a^x -w\,dx = -w\cdot x\Big|_a^x = -w(x-a)$

$V(x) = -w(x-a) + \frac{w(L-2a)}{2} = w\left(\frac{L}{2} - x\right)$

$M(x) - M(a) = \int_a^x w\left(\frac{L}{2} - x\right)dx = w\left(\frac{L}{2}x - \frac{1}{2}x^2\right)\Big|_a^x$

$M(x) = \frac{w}{2}\left(Lx - x^2 - L\cdot a + a^2\right) + \underbrace{\frac{w}{2}\left(L\cdot a - 2a^2\right)}_{M(a)} = \frac{w}{2}\left(Lx - x^2 - a^2\right)$

Check: $M(x=L-a) = \frac{w}{2}\left(L^2 - L\cdot a - (L^2 - 2L\cdot a + a^2) - a^2\right)$

$= \frac{w}{2}\left(L\cdot a - 2a^2\right) = \frac{w(L-2a)}{2}\cdot a$ OK!

x: L−a to L

$w(x) = 0$

$V(x) - V(L-a) = \int_{L-a}^x 0\,dx = 0, \quad V(x) = V(L-a) = w\left(\frac{L}{2} - (L-a)\right) = -\frac{w}{2}(L-2a)$

$M(x) - M(L-a) = \int_{L-a}^x -\frac{w}{2}(L-2a)\,dx = -\frac{w}{2}(L-2a)\cdot x\Big|_{L-a}^x = -\frac{w}{2}(L-2a)(x-(L-a))$

$M(x) = -\frac{w}{2}(L-2a)(x-L+a) + \underbrace{\frac{w}{2}(L-2a)a}_{M(L-a)} = \frac{w}{2}(L-2a)(a-x+L-a)$

$M(x) = \frac{w}{2}(L-2a)(L-x)$

Check: $M(L) = 0$ BOO YEAH!