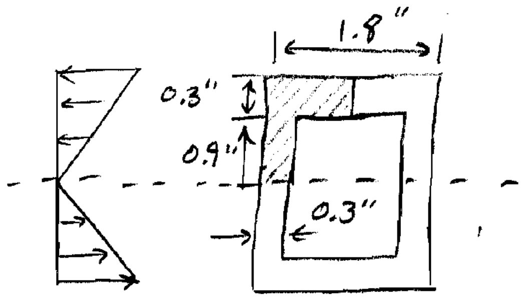
4.13 $\sigma = \dfrac{-M \cdot y}{I}$ $\qquad M = 8 \text{ k-in}$

$$I = \frac{1.8 \times 2.4^3 - 1.2 \times 1.8^3}{12}$$

$$F = \int_A \sigma \, dA$$



$$= \frac{1.8}{2}\int_{0.9}^{1.2} -\frac{M \cdot y}{I}\, dy \; +$$

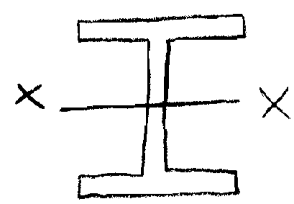
$$0.3\int_{0}^{0.9} -\frac{M \cdot y}{I}\, dy$$

$$= -\frac{M}{I}\left(\frac{1.8''}{2}\frac{y^2}{2}\Big|_{0.9''}^{1.2''} + 0.3\frac{y^2}{2}\Big|_{0''}^{0.9''}\right) = -2.17 \text{ k} \quad (\text{compression})$$

4.31 W200 × 31.3, $E = 200$ GPa

$\hookrightarrow I_x = 31.4 \,(\times 10^6 \text{ mm}^4)$

$S_x = 299 \,(\times 10^3 \text{ mm}^3)$



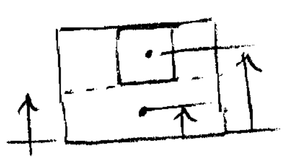
$$\frac{1}{\rho} = \kappa = \frac{M}{EI} = \frac{45 \text{ kN}\cdot\text{m}}{200 \text{ GPa} \times 31.4 \times 10^6 \text{ mm}^4} = \frac{45 \times 10^6 \text{ N}\cdot\text{mm}}{200{,}000 \frac{\text{N}}{\text{mm}^2} \times 31.4 \times 10^6}$$

$\rho = 139.6$ m

$$\sigma_{max} = \pm\frac{M \cdot c}{I_x} = \frac{M}{S_x} = \frac{45 \text{ kN}\cdot\text{m}}{299 \times 10^3 \text{ mm}^3} = 150 \text{ MPa}$$

$\hookrightarrow S_x = I_x / c$ "elastic section modulus"

4.40



$$y_{NA} = \frac{\sum E_i A_i y_i}{\sum E_i A_i} \quad \nearrow y_{NA} = 9 \text{ mm}$$

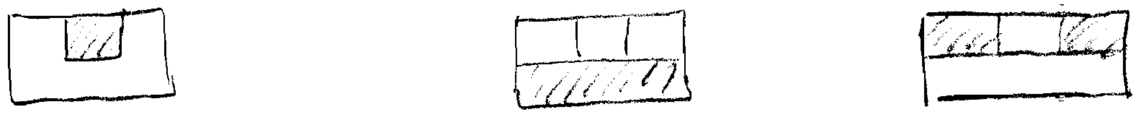
$$y_{NA} = \frac{210 \times 8 \times 8 \times 12 + 70(8 \times 24 \times 4 + 16 \times 8 \times 12)}{210 \times 8 \times 8 + 70(8 \times 24 + 16 \times 8)}$$

$$\kappa = \frac{M}{(EI)_{eff}}$$

$$\varepsilon = -\kappa \cdot y$$

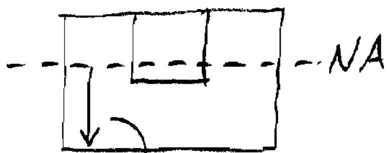
$$\sigma_i = -E_i \frac{My}{(EI)_{eff}} \qquad (EI)_{eff} = \sum E_i (I_{c_i} + A_i d_i^2)$$

$$EI = 216\left(\frac{8\times 8^3}{12} + 8\times 8\times(12-9)^2\right) + 70\left(\frac{24\times 8^3}{12} + 24\times 8\times(4-9)^2\right) + 70\left(\frac{16\times 8^3}{12} + 16\times 8\times(12-9)^2\right)$$



(a) $\sigma_{a,max} = -E_a \cdot \frac{M\cdot y}{(EI)_{eff}}$ → use max y from N.A.

+/- doesn't matter



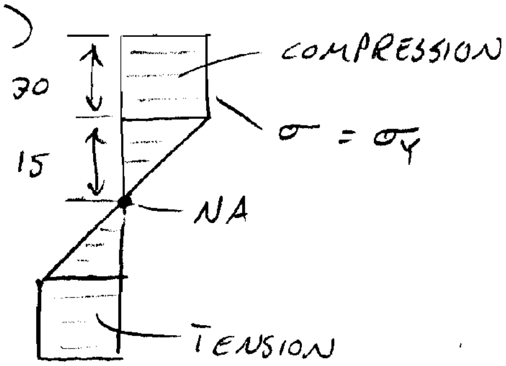
$y_{max} = -9\text{ mm}$

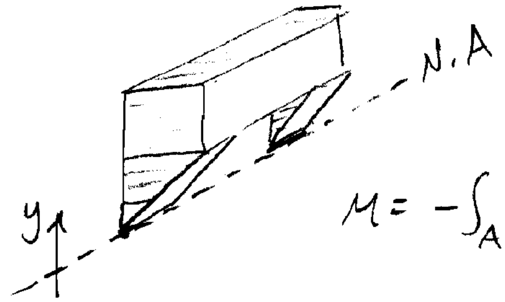
$$\sigma_{a,max} = -E_a \cdot \frac{M\cdot(-9\text{mm})}{(EI)_{eff}}$$

4.74 (a) First yield when $\sigma_{max} = \sigma_Y = \pm 240$ MPa

$$\sigma_{max} = -\frac{Mc}{I}, \quad M_Y = -\sigma_Y \cdot \frac{I}{c}$$

(b)





$$M = -\int_A y \cdot \sigma \cdot dA$$

$\sigma = -\sigma_Y \qquad 15 < y < 45$

$\sigma = -\sigma_Y \cdot \frac{y}{15} \qquad 0 < y < 15$

$$\frac{M}{2} = -60\int_{15}^{45} -\sigma_Y \cdot y\, dy - 30\int_0^{15} -\sigma_Y \cdot \frac{y}{15} \cdot y\, dy$$

$$\frac{M}{2} = 60 \cdot \sigma_Y \cdot \frac{1}{2} y^2 \Big|_{15}^{45} + 30 \cdot \frac{\sigma_Y}{15} \cdot \frac{1}{3} y^3 \Big|_0^{15}$$

$$M = 2(\underbrace{54000 + 2250}_{mm^3})\sigma_Y = 27 \times 10^6 \ N\cdot mm = 27 \ kN\cdot m$$

Shape factor : $\frac{M_u}{M_Y}$ how much extra capacity after M_Y?

Look up S, Z $\quad M_Y = S \cdot \sigma_Y \quad M_u = Z \cdot \sigma_Y$

<u>General</u>	<u>Elastic</u>
$M = -\int_A y \cdot \sigma \cdot dA$	$\sigma = -\frac{M \cdot y}{I}$
$\varepsilon = -K \cdot y$	$K = \frac{M}{EI}$
$\Sigma F = 0 = \int_A \sigma dA$ ↳ locating y	$y_{NA} = \frac{\Sigma E_i A_i y_i}{\Sigma E_i A_i}$