Practice Examination Solution

[Problem 1]

(a)

$$\tau_{max} = \frac{T \cdot (b/2)}{J_P} \quad , \quad J_P = \frac{\pi}{32}(b^4 - a^4)$$

$$\tau_{max} = \frac{32\, T \cdot (b/2)}{\pi (b^4 - a^4)} = \frac{16\, T b}{\pi (b^4 - a^4)}$$

(b) Consider a solid cylinder with the same maximum shear stress, i.e.

$$\tau_{max} = \frac{T c}{\frac{\pi}{32}(2c)^4} = \frac{2T}{\pi c^3} = \frac{16\, T b}{\pi (b^4 - a^4)}$$

$$c^3 = \frac{(b^4 - a^4)}{8b} \Rightarrow d = 2c = 2\left(\frac{b^4 - a^4}{8b}\right)^{\frac{1}{3}}$$

$$\boxed{d = \left(\frac{b^4 - a^4}{b}\right)^{1/3}}$$

[Solution 2]

$\Sigma F_x = 0$

$$\left(\sigma_{xx} + \frac{\partial \sigma_{xx}}{\partial x} dx\right)(dy \cdot 1) - (\sigma_{xx})(dy \cdot 1)$$

$$+ \left(\sigma_{yx} + \frac{\partial \sigma_{yx}}{\partial y} dy\right)(dx \cdot 1) - (\sigma_{yx})(dx \cdot 1)$$

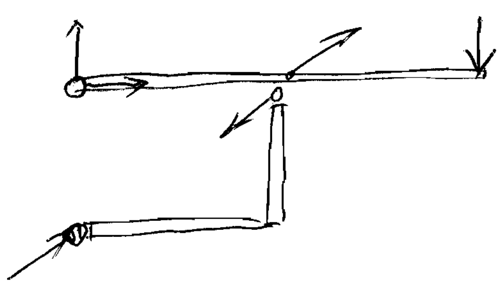
$$+ X \cdot (dx \cdot dy \cdot 1) = 0$$

$$\frac{1}{dx\,dy}\left\{ \frac{\partial \sigma_{xx}}{\partial x} dx\,dy + \frac{\partial \sigma_{yx}}{\partial y} dx\,dy + X\,dx\,dy \right\} = 0$$

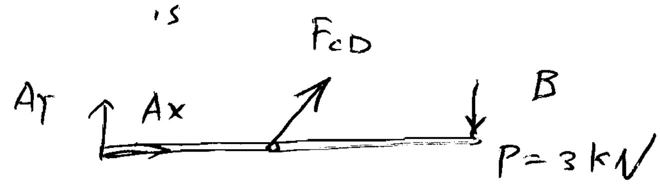
$$\Rightarrow \quad \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{yx}}{\partial y} + X = 0$$

[Solution 3]

Consider CD is a two-force member.



The free-body diagram of AB is



$\Sigma M_A = 0$ (+)

$$F_{CD} \sin 45^\circ L - 2L\,P = 0$$

$$F_{CD} = \frac{2P}{\sin 45^\circ} = 2\sqrt{2}\,P \quad \leftarrow \text{reaction in C.}$$

(Cont'd)

(Cont'd)

$\Sigma F_x = 0 \qquad A_x + F_{CD} \cos 45^\circ = 0$

$$A_x = -\frac{\sqrt{2}}{2} F_{CD} = -2P = -6\,kN$$

$\Sigma F_y = 0 \qquad A_y + F_{CD} \sin 45^\circ - P = 0$

$$A_y = P - \frac{\sqrt{2}}{2}(2\sqrt{2})P$$

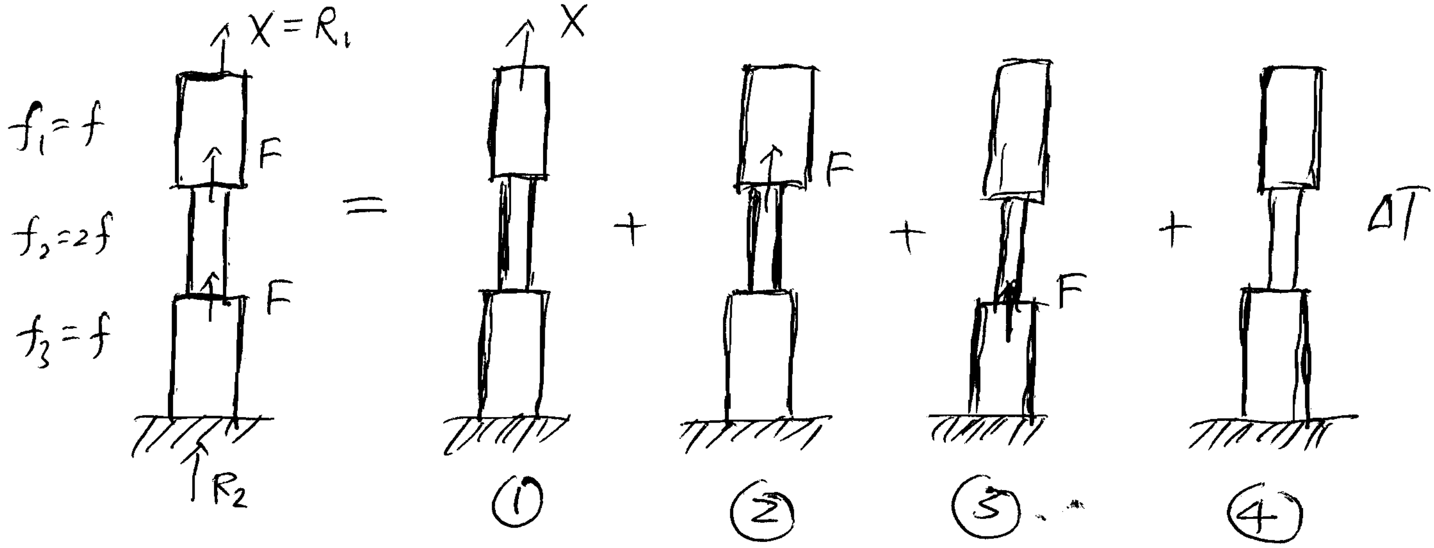
$$= -P = -3\,kN$$

And $\quad F_A = \sqrt{A_x^2 + A_y^2} = \sqrt{9 + 36} = \sqrt{45}\ kN$

reaction in A

[Problem 4] Find Reactions R_1 & R_2

Remove constraint at A and consider superposition method



$$\Delta_1 = fX + 2fX + fX = 4fX$$

$$\Delta_2 = 2fF + fF = 3fF$$

$$\Delta_3 = fF$$

$$\Delta_4 = \alpha \Delta T L + \alpha \Delta T L + \alpha \Delta T L = 3\alpha \Delta T L$$

$$\Delta = \Delta_1 + \Delta_2 + \Delta_2 + \Delta_4$$

$$= 4fX + 3fF + fF + 3\alpha \Delta T L = 0$$

R_1

$$R_1 = X = -\frac{1}{4f}(4fF + 3\alpha \Delta T L) = \boxed{-F - \frac{3L}{4f}\alpha \Delta T}$$

$\Sigma F_x = 0$;

$$R_2 + 2F + X = 0$$

$$R_2 = -2F - X = \boxed{-F + \frac{3L}{4f}\alpha \Delta T} \quad R_2$$

[Problem 5]

(a) $\sigma_Y = 0$, $\sigma_Z = \sigma_0$, find σ_X such that $\varepsilon_X = 0$

$$\varepsilon_X = \frac{\sigma_X}{E} - \nu\frac{\sigma_Y}{E} - \nu\frac{\sigma_Z}{E} = 0$$

$$\sigma_X = \nu(\sigma_Y + \sigma_Z) = \nu\sigma_0$$

(b) find the ratio σ_0/ε_Z ?

$$\varepsilon_Z = -\nu\frac{\sigma_X}{E} - \nu\frac{\sigma_Y}{E} + \frac{\sigma_Z}{E}$$

$$= -\frac{\nu^2\sigma_0}{E} + \frac{\sigma_0}{E} = \frac{1}{E}(1-\nu^2)\sigma_0$$

Therefore

$$\boxed{\frac{\sigma_0}{\varepsilon_Z} = \frac{E}{1-\nu^2}}$$