Problem 1

$\Sigma F_Y = 0 \quad \Rightarrow \quad V(x) - (V(x) + \Delta V) + q(x) \cdot \Delta X = 0$

$$V(x) = q(x)\,\Delta X \quad \Rightarrow \quad \boxed{\frac{dV}{dx} = q(x)}$$

$\Sigma M_{x+\Delta x} = 0 \ +\circlearrowleft$

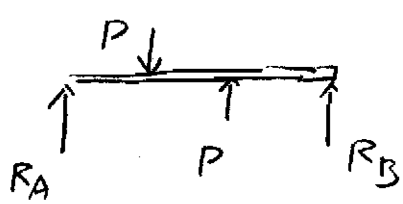
$$-M(x) + (M(x) + \Delta M) - V(x) \cdot \Delta X - (q(x) \cdot \Delta X) \cdot \frac{\Delta X}{2} = 0$$

$$\Delta M = V(x) \cdot \Delta X + q(x) \frac{\Delta X^2}{2}$$

$$\frac{\Delta M}{\Delta X} = V(x) + q(x) \cdot \frac{\Delta X}{2} \quad \Rightarrow \quad \boxed{\frac{dM}{dx} = V(x)}$$

Problem 2-a

Step 1. Find the reaction forces



$\Sigma M_B = 0 \ +\circlearrowleft$

$$-R_A \cdot L + \frac{2}{3} L P - \frac{L}{3} P = 0$$

$$\boxed{R_A = \frac{P}{3}}$$

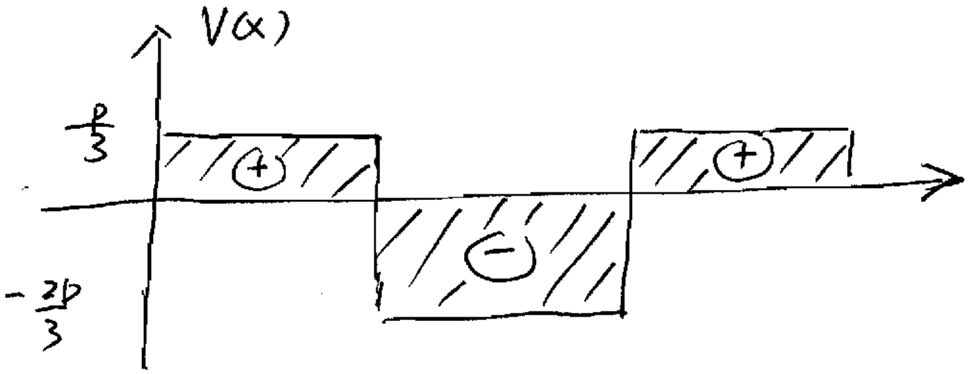
By symmetry : $\boxed{R_B = -\frac{P}{3}}$

$$\boxed{+w(x) = P\langle x-\tfrac{L}{3}\rangle_*^{-1} - P\langle x-\tfrac{2L}{3}\rangle_*^{-1}}$$

step 2 $\quad \frac{dV}{dx} = EI\, y'''(x) = -w(x) \quad ; \quad V(0) = R_A = \frac{P}{3}$

$$V(x) - V(0) = -P\langle x-\tfrac{L}{3}\rangle^0 + P\langle x-\tfrac{2L}{3}\rangle^0$$

$$\boxed{V(x) = \frac{P}{3} - P\langle x-\tfrac{L}{3}\rangle^0 + P\langle x-\tfrac{2L}{3}\rangle^0}$$

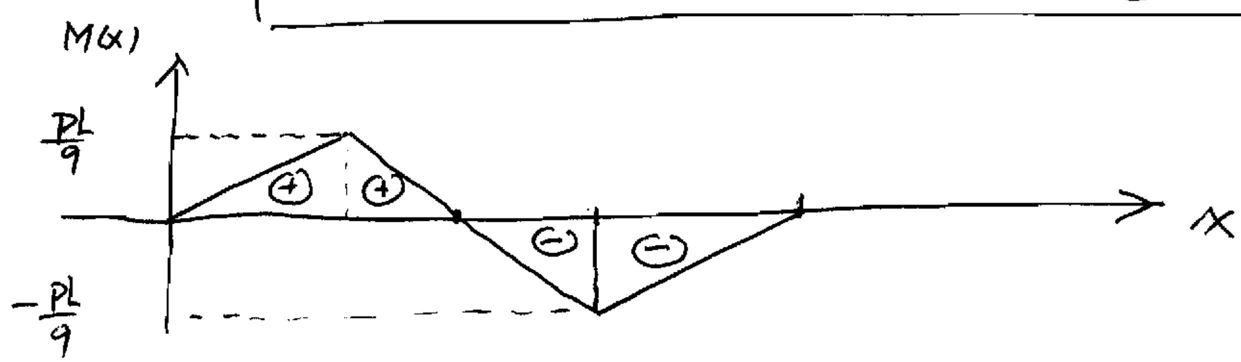


step 3 $\quad \frac{dM}{dx} = V(x) = \frac{P}{3} - P\langle x-\tfrac{L}{3}\rangle^0 + P\langle x-\tfrac{2L}{3}\rangle^0$

$$M(x) - M(0) = \frac{P}{3}x - P\langle x-\tfrac{L}{3}\rangle + P\langle x-\tfrac{2L}{3}\rangle$$

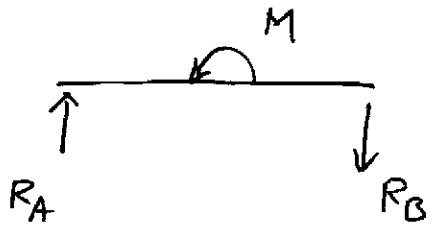
$$M(0) = M_A = 0$$

$$\boxed{M(x) = \frac{P}{3}x - P\langle x-\tfrac{L}{3}\rangle + P\langle x-\tfrac{2L}{3}\rangle}$$



Problem 2-b

Step 1



$\Sigma F_Y = 0 \Rightarrow R_A = R_B$

$\Sigma M = 0$

$R_A \cdot L = M$

$R_A = R_B = \frac{M}{L}$

Step 2

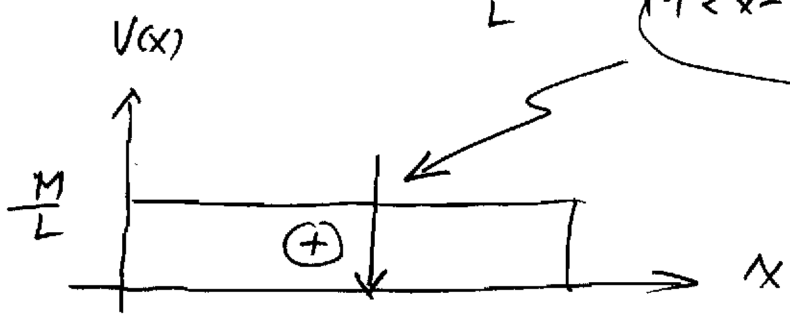
$$W(x) = M \langle x - \tfrac{L}{2} \rangle_*^{-2}$$

$$\frac{dV}{dx} = -W(x) = -M \langle x - \tfrac{L}{2} \rangle^{-2}$$

$$V(x) = -M \langle x - \tfrac{L}{2} \rangle^{-1} + V(0)$$

$$= \frac{M}{L} - M \langle x - \tfrac{L}{2} \rangle_*^{-1}$$

You don't need to draw it part



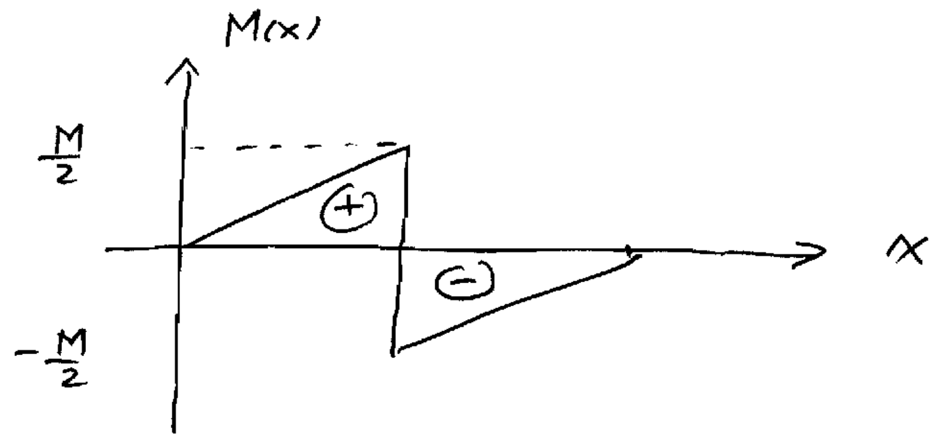
Step 3

$$\frac{dM}{dx} = V(x) = \frac{M}{L} - M \langle x - \tfrac{L}{2} \rangle_*^{-1}$$

$$M(x) - M(0) = \frac{M}{L} x - M \langle x - \tfrac{L}{2} \rangle^{0}$$

$M_A = M(0) = 0$

$$\Rightarrow \boxed{M(x) = \frac{M}{L} x - M \langle x - \tfrac{L}{2} \rangle^{0}}$$



Problem 3 — Maximum shear stress occurs at $y=0$

step 1. Find I_z & $Q(0)$

$$I_z = \frac{(4t)(4t)^3}{12} - \frac{(2t)(2t)^3}{12} = \frac{t^4}{12}(4^4 - 2^4)$$

$$= 20t^4$$

$$Q(0) = 2 \cdot t \cdot (t \times 2t) + t \cdot (2t) \cdot \frac{3}{2}t$$

$$= 4t^3 + 3t^3 = 7t^3$$

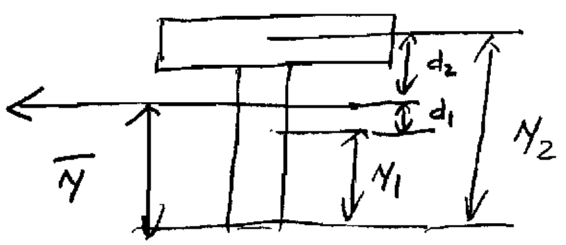
step 2 What is "t" in $\tau = \frac{VQ}{I_z t}$?

"t" = 2t !

$$\boxed{\tau = \frac{V \cdot (7t^3)}{(20t^4)(2t)} = \frac{7}{40}\frac{V}{t^2}}$$

Problem 4

(a) Position of centroidal axis ?



	y_i	A_i	$y_i A_i$
①	$h/2$	hb	$\frac{1}{2}h^2 b$
②	$h+\frac{b}{2}$	hb	$h^2 b+\frac{1}{2}hb^2$
		$2hb$	$\frac{3}{2}h^2 b+\frac{1}{2}hb^2$

$$\bar{y} = \frac{\frac{3}{2}h^2 b + \frac{1}{2}hb^2}{2hb} = \frac{1}{4}(3h+b)$$

(b) I_z ?

$$d_1 = \bar{y} - y_1 = \frac{1}{4}(3h+b) - \frac{h}{2} = \frac{h}{4} + \frac{b}{4} = \frac{1}{4}(h+b)$$

$$d_2 = y_2 - \bar{y} = \left(h+\frac{b}{2}\right) - \frac{1}{4}(3h+b) = \frac{1}{4}(h+b)$$

$$I_z = I_1 + I_2$$

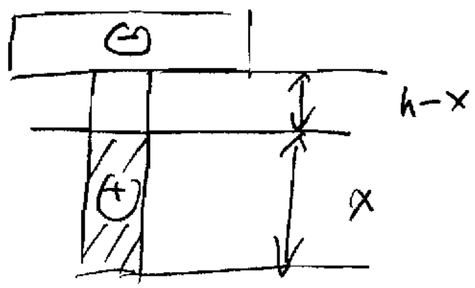
$$I_1 = \frac{bh^3}{12} + d_1^2 A_1 = \frac{bh^3}{12} + \frac{1}{16}(h+b)^2 \cdot bh$$

$$I_2 = \frac{hb^3}{12} + d_2^2 A_2 = \frac{hb^3}{12} + \frac{1}{16}(h+b)^2 \cdot bh$$

$$I_z = \frac{bh}{12}(h^2+b^2) + \frac{bh}{8}(h+b)^2 = \frac{bh}{24}(5h^2+5b^2+3hb)$$

(c) Inelastic neutral axis ?

$$\Sigma F_x = 0$$

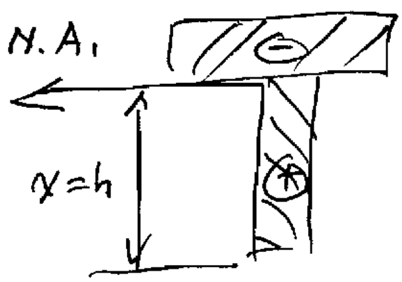


$$\sigma_Y^T \cdot (x \cdot b) - |\sigma_Y^C| (b(h-x) + bh) = 0$$

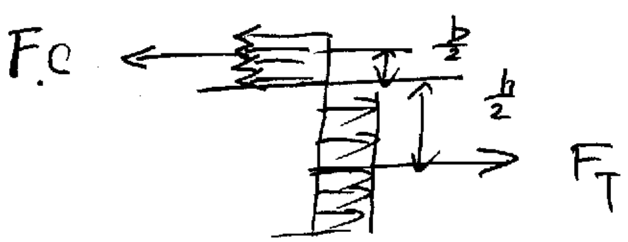
Since $\sigma_Y^T = |\sigma_Y^C| = 100\ MPa$

$$\Rightarrow \quad 2bx - 2bh = 0$$

$$\Rightarrow \quad \boxed{x = h} \longleftarrow \text{The position of inelastic N.A.}$$



(d) Mult ?



$$M_{ult} = |\sigma_Y^C| \cdot A_c \cdot \frac{b}{2} + (\sigma_Y^T) \cdot A_T \cdot \frac{h}{2}$$

$$= |\sigma_Y| A_c \left(\frac{b}{2} + \frac{h}{2}\right)$$

Since $|\sigma_T^C| = |\sigma_Y^T| = 100\ MPa$

and $A_c = A_T = b \cdot h$

$$M_{ult} = \frac{100\ MPa}{2} \cdot bh\,(h+b)$$

$$\boxed{M_{ult} = 50 \times 10^6\ bh\,(h+b)}$$

5

$$w = M\langle x-a\rangle^{-2}$$

$$EI\frac{d^4y}{dx^4} = -M\langle x-a\rangle^{-2}$$

B.C. $V(0) = 0$, $M(0) = 0$

$EIy'(a+b) = 0$, $EIy(a+b) = 0$

Step 1

$$EIy''' = \cancelto{0}{V(0)} - M\langle x-a\rangle^{-1}$$

$$EIy'' = \cancelto{0}{M(0)} - M\langle x-a\rangle^{0}$$

$$EIy' = -M\langle x-a\rangle + C_3$$

$$EIy(x) = -\frac{M}{2}\langle x-a\rangle^2 + C_3x + C_4$$

Consider $EIy'(a+b) = 0$

$$-Mb + C_3 = 0 \qquad C_3 = Mb$$

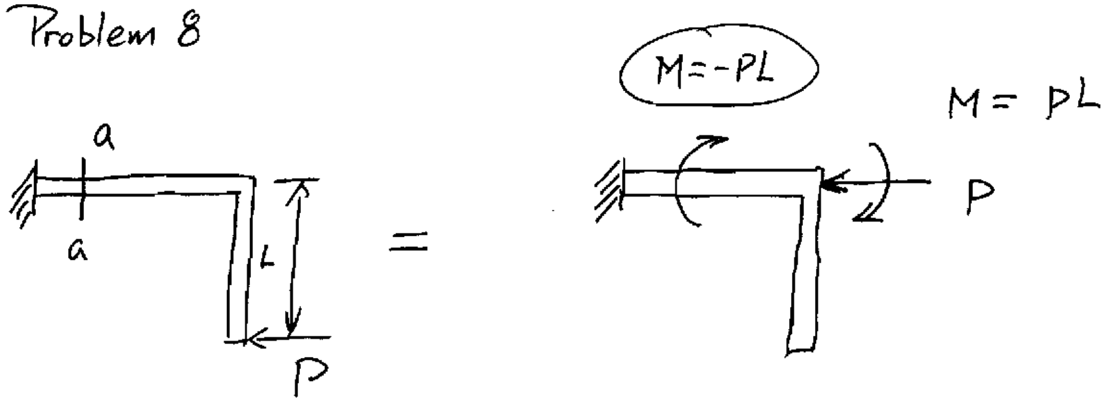
$Ey(a+b) = 0$

$$-\frac{M}{2}b^2 + Mb\cdot b + C_4 = 0$$

$$C_4 = -\frac{Mb^2}{2}$$

$$\boxed{EIy(x) = -\frac{M}{2}\langle x-a\rangle^2 + Mbx - \frac{Mb^2}{2}}$$

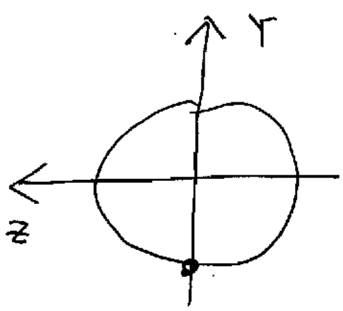
Problem 8



Step 1

$$\sigma_x = -\frac{P}{A} - \frac{(-PL)\cdot y}{I_z}, \qquad I_z = \frac{\pi}{4}R^4, \quad R=0.05$$

Step 2



At the point A

$y = -R$

$$\sigma_x = \frac{-100\times10^3}{\pi\,(0.05)^2} + \frac{100\times10^3 \times 10 \times (-0.05)}{\frac{1}{4}\pi\,(0.05)^4}$$

$$= -\frac{100\times10^3}{\pi\,(0.05)^2}\,(1+800) = 1.01986\times10^{10} \quad \text{Pa}$$