UNIVERSITY OF CALIFORNIA, BERKELEY
FALL SEMESTER 2002

FINAL EXAMINATION

(CE130-1 Mechanics of Materials)

Problem 1: (10 points)

A pin-jointed 3-column structure is shown in the Figure 1. There is an external force, P, acting on the point C. When external force P is below a critical load, P_{cr}, the position shown in the figure is the initial equilibrium position. (1) What are the internal transverse forces for column AC, BC, and CD when $P < P_{cr}$, why ? (2) What are the internal moment for column AC, BC, and CD when $P < P_{cr}$, why ? (3) Find internal axial forces for column AC, BC, and CD; (4) Find the critcal load P_{cr} ?

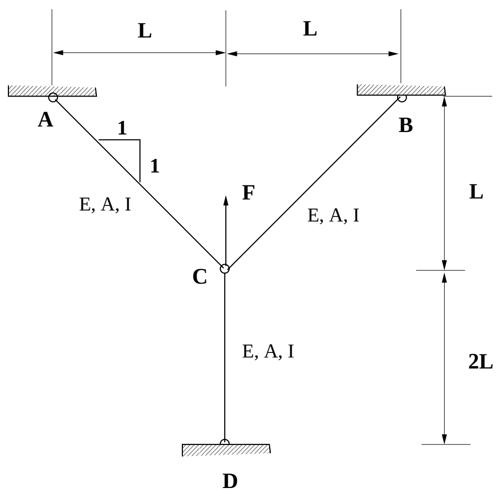

Figure 1: Schematic illustration of problem 1

(Hint: (1) use Castigliano's second theorem, the energy for axially deformed column is, $U = \dfrac{P^2 L}{2EA}$, where P is the axial force, L is the length of the column, E is the Young's modulus, and A is the cross section of the column; (2) Euler's formula $P_{cr} = \pi^2 EI/L^2$.)

Problem 2 (10 points)

A simply supported beam subjected two concentrated forces that have the same magnitude, P, shown in Figure 2. Draw shear and moment diagrams. (10 points)

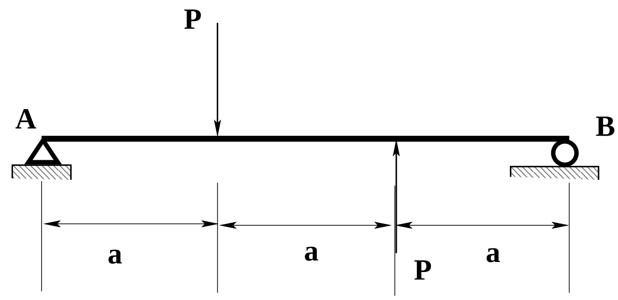

Figure 2: Simply supported beam with concentrated forces.

Problem 3: (20 points)

Consider a plane stress state as follows

$$\boldsymbol{\sigma} = \begin{pmatrix} 5 & 3 \\ 3 & -3 \end{pmatrix} \qquad \text{(MPa)}$$

A. draw Mohr's circle of the stress state at that point;

B. find principal stresses σ_1, σ_2, and show the results on properly oriented element in physical space;

C. find the maximum shear stress, and show the results on properly oriented element in physical space;

D. find at least one angle between the right face of the initial infinitesimal element and the planes on which the normal stress is zero.

(Hint:

$$\begin{aligned}
\sigma_1 &= \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \\
\sigma_2 &= \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \\
tan 2\theta_p &= \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} \\
tan 2\theta_s &= -\frac{(\sigma_x - \sigma_y)/2}{\tau_{xy}}
\end{aligned}$$

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Problem 4 (10 points)

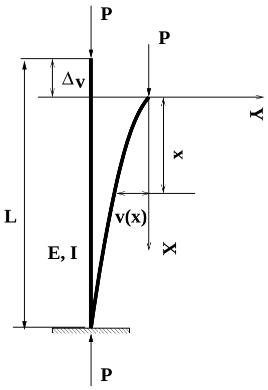

Figure 3: Problem 4

Determine the critical buckling load P_{cr} for a cantilever elastic column with span L and constant stiffness (rigidity) EI.

Use any methods that you feel comfortable with.

(Hint:

(1) Make a cut at the cross section X; Draw free-body diagram for the isolated part, and derive the second order differential equation that governs the stability of a cantilever beam, and find the critical load;

or

(2) Use the fourth order differential equation

$$\frac{d^4v}{dx^4} + \lambda^2 \frac{d^2v}{dx^2} = 0 \tag{1}$$

where $\lambda^2 = \dfrac{P}{EI}$. Write down the boundary conditions, solve the differential equation, and find the critical load. The general form of homogeneous solution of Eq. (1) is $v(x) = C_1 \cos \lambda x + C_2 \sin \lambda x + C_3 x + C_4$.

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Problem 5 (10 points)

Consider an infinitesimal element shown in Figure 4. The normal stresses and shear stresses on two oblique planes are given. Find σ_x and τ_{xy}.

Problem 6 (20 points)

A box beam is made by nailing together four boards in the configurations shown in Figure 5 and labeled as *Config.1* and *Config.2*. The beam supports a concentrated load of 1000 N at its midspan, and it rests on simple supports as shown in Figure 5. Assume each nail can withstand an allowable shear force of 200 N.

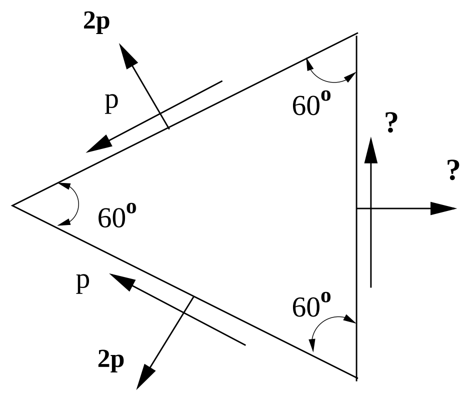

Figure 4: Problem 5

a . draw shear diagram;

b . what is the maximum spacing (Δ_s) for configuration 1;

c . what is the maximum spacing (Δ_s) for configuration 2;

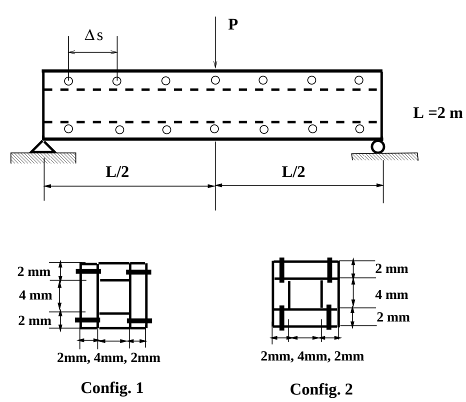

Figure 5: Problem 6

(Hint:

$$
\begin{aligned}
q &= \frac{VQ}{I_z}; \\
Q &= \int_A y dA = \bar{y}A \\
q &= \frac{N_{allowable}}{\Delta_s}
\end{aligned}
$$

where $N_{allowable}$ stands for allowable shear force by the nails.)

Problem 7 (10 points)

A planar frame ABCD is subjected a concentrated moment, M, at the end point D as shown in Figure 6.

(a) draw moment diagram along the frame;

(b) find the cross section rotation at point D, i.e. θ_D ?

(Hint:

(1) use virtual force method,

$$\bar{1} \times \theta_D = \int \bar{m}_c(s) \frac{M(s)}{EI} ds$$

where s is a local coordinate;

or (2) use Castigliano's second theorem,

$$\theta_D = \frac{\partial U^*}{\partial M} = \frac{\partial U}{\partial M}$$

$$U = \frac{1}{2EI} \int_0^L M(s)^2 ds$$

)

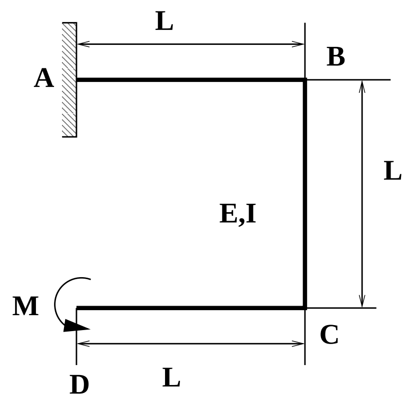

Figure 6: Problem 7

Problem 8 (10 points)

A L-shaped beam is made of a rectangular section and a solid cylinder section with radius $R = 0.1m$. The span of the both section is $L = 2.0\ m$. There is a concentrated load, $P = 300N$, acting on the free-end of the rectangular (as shown in Figure 7.). (1) Draw the moment diagram, shear diagram, and internal torque diagram; (2) Find the normal stress σ_x, shear stresses τ_{xy} and

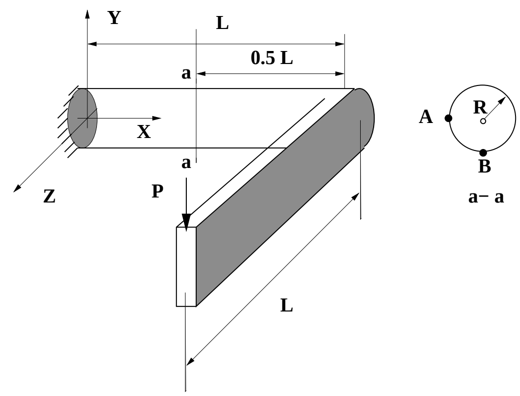

Figure 7: Problem 8

τ_{xz} at point A; (3) Find the normal stress σ_x and shear stress τ_{xy} and τ_{xz} at point B;

Hints:

$$
\begin{aligned}
\sigma_x &= -\frac{M_z y}{I_z} \\
\tau &= \frac{VQ(y)}{I_z t} \\
\tau &= \frac{T\rho}{I_p} \\
I_z &= \frac{1}{2} I_p = \frac{R^4 \pi}{4} \\
\text{For semi} - \text{circle,} & \\
Q(y)\Big|_{y=0} &= \frac{2}{3} R^3
\end{aligned}
$$