UNIVERSITY OF CALIFORNIA, BERKELEY College of Engineering

Advanced Mechanics of Materials (C131)
HOMEWORK I (due on next Friday)

Problem I-1

Simplify each of the following expressions by employing the summation property of the Kronecker delta. Perform summations over the range 1,2,3 for (e) and (f).

$$\text{(a) } \delta_{ij}\delta_{jn}; \quad \text{(b) } a_{ij}\delta_{in}; \quad \text{(c) } \delta_{ij}\delta_{jn}\delta_{ni}; \tag{1}$$

$$\text{(e) } \delta_{ij}\delta_{ij}; \quad \text{(f) } \delta_{ii}, \quad \text{(g) } A_i B_j \delta_{ij} - B_m A_n \delta_{mn} \ . \tag{2}$$

Problem I-2

Consider two Cartesian coordinates with basis $\{\mathbf{e}_i\}$ and $\{\mathbf{e}'_j\}$, i,j =1,2,3. Let,

$$\ell_1 := \cos(\mathbf{e}'_1, \mathbf{e}_1), \quad m_1 := \cos(\mathbf{e}'_1, \mathbf{e}_2), \quad n_1 := \cos(\mathbf{e}'_1, \mathbf{e}_3), \tag{3}$$

$$\ell_2 := \cos(\mathbf{e}'_2, \mathbf{e}_1), \quad m_2 := \cos(\mathbf{e}'_2, \mathbf{e}_2), \quad n_2 := \cos(\mathbf{e}'_2, \mathbf{e}_3), \tag{4}$$

$$\ell_3 := \cos(\mathbf{e}'_3, \mathbf{e}_1), \quad m_3 := \cos(\mathbf{e}'_3, \mathbf{e}_2), \quad n_3 := \cos(\mathbf{e}'_3, \mathbf{e}_3), \tag{5}$$

where $\cos(\mathbf{e}'_i, \mathbf{e}_j)$ denotes cosion of the angle between $\mathbf{e}'_i$ and $\mathbf{e}_j$.

Show

$$\ell_i^2 + m_i^2 + n_i^2 = 1, \ i = 1, 2, 3 \tag{6}$$

and

$$\begin{aligned} \ell_1\ell_2 + m_1 m_2 + n_1 n_2 &= 0 \\ \ell_2\ell_3 + m_2 m_3 + n_2 n_3 &= 0 \\ \ell_1\ell_3 + m_1 m_3 + n_1 n_3 &= 0 \end{aligned} \tag{7}$$

Problem I-3

Show

$$[\beta_{ij}] = \begin{bmatrix} \frac{12}{25} & \frac{3}{5} & -\frac{16}{25} \\ -\frac{9}{25} & \frac{4}{5} & \frac{12}{25} \\ \frac{4}{5} & 0 & \frac{3}{5} \end{bmatrix} \tag{8}$$

is orthogonal, that is, $[\beta_{ik}][\beta^T_{kj}] = [\beta^T_{ik}][\beta_{kj}] = [\delta_{ij}]$.

Problem I-4

For the matrix

$$a_{ij} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 2 \\ 0 & 2 & 3 \end{bmatrix} \tag{9}$$

calculate the values of

$$
\begin{array}{ll}
(a) & a_{ii}, \\
(b) & a_{ij}a_{ij} \\
(c) & a_{ij}a_{ik},\ \text{when } j=1, k=1,\ \text{and when } j=1, k=2
\end{array}
$$

Problem I-5

Assume the coordinate transformation matrix between old coordinate $\{\mathbf{e}_i\}$ and the new coordinate $\{\mathbf{e}_i^{'}\}$ is,

$$
[Q_{ij}] = \begin{bmatrix} 1/2 & -\sqrt{3}/2 & 0 \\ \sqrt{3}/2 & 1/2 & 0 \\ 0 & 2 & 1 \end{bmatrix} \tag{10}
$$

and the Cauchy stress in the old coordinate system is

$$
[\sigma_{ij}] = \begin{bmatrix} 10 & 1 & 0 \\ 1 & 5 & 1 \\ 0 & 1 & 1 \end{bmatrix} \tag{11}
$$

Find the Cauchy stress components in the new coordinate, i.e. $[\sigma_{ij}^{'}]$.

Problem I-6 Write the following governing equation of hydrodynamics (a special form of Navier-Stokes equation) in terms of indicial notation,

$$
\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot grad)\mathbf{v} = \mathbf{f} - \frac{1}{\rho} grad\ p + \nu \Delta \mathbf{v} \tag{12}
$$

where $\mathbf{u}$ is the velocity vector, p is the pressure (scalar), $\mathbf{f}$ is the body force vector, ρ is the density of the fluid, and ν is the viscosity.

The differential operators,

$$
grad := \frac{\partial}{\partial x_i}\mathbf{e}_i, \quad \text{and} \quad \Delta := \frac{\partial^2}{\partial x_i^2}\ . \tag{13}
$$