UNIVERSITY OF CALIFORNIA, BERKELEY College of Engineering

Advanced Mechanics of Materials (C131)
HOMEWORK II (due on next Friday)

Problem II-1 Given stress tensor at one point,

$$[\sigma_{ij}] = \begin{bmatrix} 100 & 50 & 600 \\ 50 & 200 & 300 \\ 600 & 300 & -300 \end{bmatrix} \quad MP_a \tag{1}$$

Evaluate **1**.$\sigma_{ij}\delta_{ij}$, **2**.$\sigma_{3j}\delta_{j3}$, **3**.$\sigma_{ij} - \frac{1}{3}\sigma_{mm}\delta_{ij}$.

Problem II-2
Consider a vector field, $\mathbf{V} = 3x_1\mathbf{e}_1 + 2x_1x_2^2\mathbf{e}_2 + 3x_1x_3^2\mathbf{e}_3$. Calculate,
1. $V_{i,i}$, **2.** $V_{i,i2}$, and **3.** $V_{j,3j}$.

Problem II-3(Ugural-Fenster I.8)
For what body forces will the following stress field describe a state of equilibrium ?

$$\begin{array}{ll} \sigma_{xx} = -2x^2 + 3y^2 - 5z & \sigma_{xy} = z + 4xy - 7 \\ \sigma_{yy} = -2y^2 & \sigma_{xz} = -3x + y + 1 \\ \sigma_{zz} = 3x + y + 3z - 5 & \sigma_{yz} = 0 \quad . \end{array} \tag{2}$$

Problem II-4(Ugural-Fenster I.30)
The state of stress at a point in an x, y, z coordinate systen is

$$[\sigma_{ij}] = \begin{bmatrix} 20 & 12 & -15 \\ 12 & 0 & 10 \\ -15 & 10 & 6 \end{bmatrix} MP_a \tag{3}$$

Determine the stress tensor relative to an new coordinate system that is defined by rotating x and y coordinate through an angle of 30^o counterclockwise about the z-axis.

Problem II-5(Ugural-Fenster I.36)
The stresses(MP_a) with respect to an $\{x_i\}$ coordinate system are described by

$$\begin{array}{ll} \sigma_{11} = x_1^2 + x_2 & \sigma_{22} = -x_1 + 6x_2 + x_3 \\ \sigma_{33} = x_2^2 - 5 & \sigma_{12} = \sigma_{13} = \sigma_{23} = 0 \end{array}$$

At point $(3, 1, 5)$, determine (a) $[\sigma'_{ij}]$ if

$$\ell_1 = 1, \quad m_2 = \frac{1}{2}, \quad n_2 = \frac{\sqrt{3}}{2}, \quad m_3 = -\frac{\sqrt{3}}{2} \tag{4}$$

and (b) $[\sigma'_{ij}]$ if

$$\ell_1 = \frac{2}{\sqrt{5}}, \quad m_1 = -\frac{1}{\sqrt{5}}, \quad \textit{and } n_3 = 1 \ . \tag{5}$$

Problem II-6(Ugural-Fenster I.37)
Determine $[\sigma'_{ij}]$ if

$$[\sigma_{ij}] = \begin{bmatrix} 12 & 6 & 9 \\ 6 & 10 & 3 \\ 9 & 3 & 14 \end{bmatrix} MP_a \tag{6}$$

and

$$\ell_1 = \tfrac{1}{2} \qquad \ell_2 = -\tfrac{\sqrt{3}}{2} \qquad \ell_3 = 0$$

$$m_1 = \tfrac{\sqrt{3}}{2} \qquad m_2 = \tfrac{1}{2} \qquad m = 0$$

$$n_1 = 0 \qquad n_2 = 0 \qquad n_3 = 1$$

Problem II-7(Ugural-Fenster I.42)
The state of stress at a point in a member relative to an x, y, z coordinate system is

$$[\sigma_{ij}] = \begin{bmatrix} 20 & 10 & -10 \\ 10 & 30 & 0 \\ -10 & 0 & 50 \end{bmatrix} MP_a \tag{7}$$

Determine the normal stress, $\sigma^{(n)}$, and the shear stress, $\tau^{(n)}$, on the surface intersecting the point and parallel to the plane: $2x + y - 3z = 9$.

Problem II-8
The state of stress at a point in a member relative to an x_1, x_2, x_3 coordinate system is

$$[\sigma_{ij}] = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -2 \\ 0 & -2 & 0 \end{bmatrix} MP_a \tag{8}$$

Find the principal stresses and principal directions.