

Advanced Mechanics of Materials (C131)
HOMEWORK III (due on next Friday)

Problem III-1

The Cauchy stress tensor at one point is,

$$\begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & \sqrt{3} \\ 0 & \sqrt{3} & 2 \end{bmatrix}, \quad MP_a \quad (1)$$

1. Find all three principal stresses; **2.** Assume the normal of an inclined plane forms an angle $\alpha = 45^\circ$ with principal axis 1, and an angle $\gamma = 45^\circ$ with principal axis 3. Use 3D Mohr's circle find the normal stress and shear stress on the inclined plane.

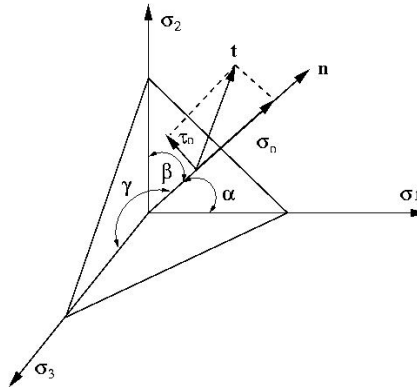


Figure 1: Problem I

Problem III-2(Ugural-Fenster I.59)

Given the principal stresses σ_1, σ_2 and σ_3 at a point in an elastic solid, prove that the maximum shearing stress at the point always exceeds the octahedral shearing stress.

Problem III-3 The principal stresses at a point in a structural member are $\sigma_1 = 20 MP_a$, $\sigma_2 = 15 MP_a$, and $\sigma_3 = -10 MP_a$. Use Mohr's circle to determine the normal and shear stresses on an inclined plane defined by $\alpha = 45^\circ$ and $\gamma = 45^\circ$.

Problem III-4(Ugural-Fenster I.66)

The principal stresses at a point in a solid are $\sigma_1 = 56 MP_a$, $\sigma_2 = 35 MP_a$, and $\sigma_3 = 14 MP_a$. Using Mohr's circle, obtain (a) the maximum shearing stress and (b) the normal and shearing stresses on the octahedral planes.

Problem III-5

Consider an infinitesimal tetrahedron as shown in Fig. 1(a). There is an oblique plane whose normal \mathbf{n} forms angles α, β and γ with the three principal axes.

Show that in Fig. 1(b) the length of the line segment, $\overline{OC} = |\mathbf{t}|$, and the length of horizontal segment $\overline{OC_1} = \sigma_n$, and the length of vertical segment, $\overline{CC_1} = \tau_n$.

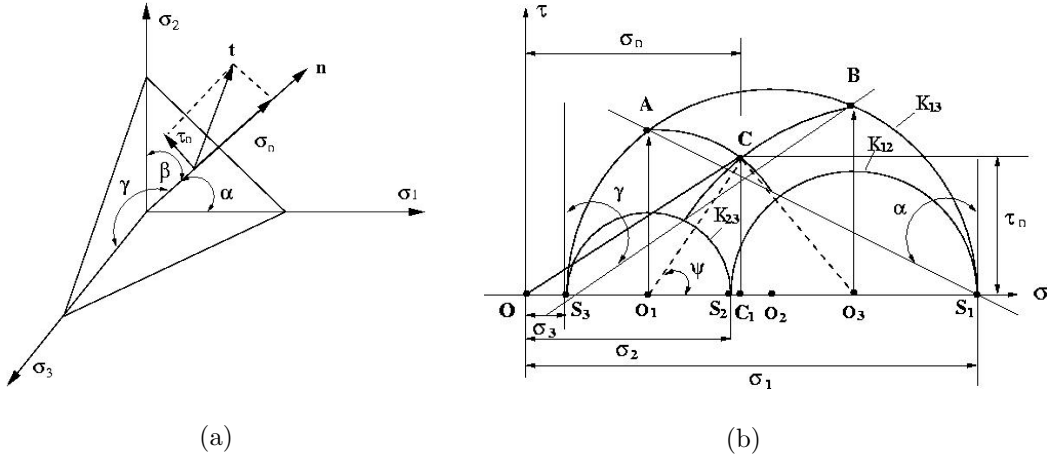


Figure 2: How to find normal stress/shear stress by using 3D Mohr's circle.

Problem III-6

At a point in a loaded member, the stresses relative to an x, y, z coordinate system are given by

$$[\sigma_{ij}] = \begin{bmatrix} 6 & 0 & 1 \\ 0 & -4 & 0 \\ 1 & 0 & 3 \end{bmatrix} \text{ MP}_a \quad (2)$$

Calculate the magnitude and direction of principal stresses.