UNIVERSITY OF CALIFORNIA, BERKELEY College of Engineering

Advanced Mechanics of Materials (C131)
HOMEWORK V (due on next Friday)

Problem V-1

The state of stress at a point is

$$\boldsymbol{\sigma} = \begin{bmatrix} 7 & 2 & 0 \\ 2 & 5 & 0 \\ 0 & 0 & -3 \end{bmatrix} MP_a \tag{1}$$

Decompose this stress tensor into the dilatational stress tensor and devatoric stress tensor. Find the values of principle deviatoric stress tensor, and find the normal deviatoric stress on the octahedral plane.

Problem V-2(Ugural-Fenster II.44)

A solid bronze sphere ($E = 110GP_a, \nu = \frac{1}{3}$, $r = 150mm$) is subjected to hydrostatic pressure p so that its volume is reduced by 0.5%. Determine (a) the pressure p, and (b) the strain energy U stored in the sphere. (Note: volume of a sphere $V = \frac{4}{3}\pi r^3$.)

Problem V-3

Show that for the case of plane strain, in the absence of body forces, the Navier equations are as follows:

$$\nabla^2 u + \frac{1}{1-2\nu}\frac{\partial}{\partial x}\Big(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\Big) = 0 \tag{2}$$

$$\nabla^2 v + \frac{1}{1-2\nu}\frac{\partial}{\partial y}\Big(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\Big) = 0 \tag{3}$$

Problem V-4(Ugural-Fenster III.2) A stress distribution is given by

$$\begin{aligned} \sigma_{xx} &= pyx^3 - 2c_1xy + c_2y \\ \sigma_{yy} &= pxy^3 - 2px^3y \\ \sigma_{xy} &= -\frac{3}{2}px^2y^2 + c_1y^2 + \frac{1}{2}px^4 + c_3 \end{aligned}$$

where p and c's are constants. (a) Verify that this stress field represents a solution for a thin plate of thickness t (Fig. 3); (b) obtain the corresponding stress function; (c) find the resultant normal and shearing boundary force (traction forces, t_x, t_y) along edges $y = 0$ and $y = b$ of the plate; (d) Determine the c's so that edges $x = \pm a$ are free of shearing stress and no normal stress acts on edge $x = a$.

Problem V-5(Ugural-Fenster III.5)

Determine whether the following stress functions satisfy the conditions of compatibility for a two-dimensional problem:

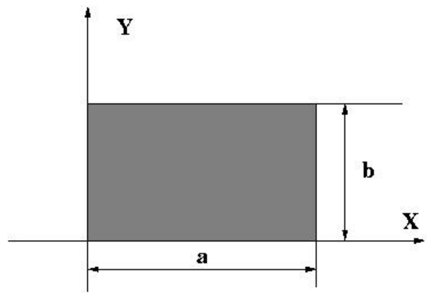

Figure 1: Problem 4

$$\Phi_1 = ax^2 + bxy + cy^2 \tag{4}$$
$$\Phi_2 = ax^3 + bx^2y + cxy^2 + dy^3 \tag{5}$$

Here a, b, c, and d are constants. Also obtain the stress fields that arise from Φ_1 and Φ_2.

Problem V-6(Ugural-fenster III.15)

Figure P3.15 shows a thin cantilever beam of unit thickness carrying a uniform load of intensity p per unit length. Assume that the stress function is expressed by

$$\Phi = ax^2 + bx^2y + cy^3 + dy^5 + ex^2y^3 \tag{6}$$

in which $a \cdots e$ are constants. Determine (a) the requirements on $a, \cdots, e$ so that Φ is biharmonic; (b) the stresses σ_x, σ_y and σ_{xy}.