CE 131 HW1

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problem I-1   note: (d) is missing in problem sheet.

a) $\delta_{ij}\delta_{in} = \delta_{in} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \delta_{ij}$

b) $a_{ij}\delta_{in} = a_{nj} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{32} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

c) $\delta_{ij}\delta_{jn}\delta_{ni} = \delta_{ij}\delta_{ji} = \delta_{ii} = \delta_{11} + \delta_{22} + \delta_{33} = 3$

d) missing in problem sheet

e) $\delta_{ij}\delta_{ij} = \delta_{ii} = 3$

f) $\delta_{ii} = \delta_{11} + \delta_{22} + \delta_{33} = 3$

g) $A_i B_j \delta_{ij} - B_m A_m = A_i B_i - B_m A_m$
As m is dummy index, $m \rightarrow i$
$= A_i B_i - B_i A_i = 0$

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Problem 2

$$\underline{e}_i' = (\underline{e}_i' \cdot \underline{e}_1)\underline{e}_1 + (\underline{e}_i' \cdot \underline{e}_2)\underline{e}_2 + (\underline{e}_i' \cdot \underline{e}_3)\underline{e}_3$$

$$= l_i \underline{e}_1 + m_i \underline{e}_2 + n_i \underline{e}_3$$

$$\underline{e}_i' \cdot \underline{e}_j' = (l_i \underline{e}_1 + m_i \underline{e}_2 + n_i \underline{e}_3) \cdot (l_j \underline{e}_1 + m_j \underline{e}_2 + n_j \underline{e}_3)$$

$$= \boxed{l_i l_j + m_i m_j + n_i n_j = \delta_{ij}} \quad \text{— ①}$$

From ①,

$$\Rightarrow \delta_{11} = l_1 l_1 + m_1 m_1 + n_1 n_1 = 1$$
$$\delta_{22} = l_2 l_2 + m_2 m_2 + n_2 n_2 = 1$$
$$\delta_{33} = l_3 l_3 + m_3 m_3 + n_3 n_3 = 1$$

$$\therefore l_i^2 + m_i^2 + n_i^2 = 1 \quad (i = 1, 2, 3)$$

From ①, $i \neq j \rightarrow$

$$l_1 l_2 + m_1 m_2 + n_1 n_2 = 0$$
$$l_1 l_3 + m_1 m_3 + n_1 n_3 = 0$$
$$l_2 l_3 + m_2 m_3 + n_2 n_3 = 0.$$

problem I-3

$$[\beta_{ik}][\beta_{kj}]^T = \begin{bmatrix} \frac{12}{25} & \frac{3}{5} & -\frac{16}{25} \\ -\frac{9}{25} & \frac{4}{5} & -\frac{12}{25} \\ -\frac{4}{5} & 0 & -\frac{3}{5} \end{bmatrix} \begin{bmatrix} \frac{12}{25} & -\frac{9}{25} & -\frac{4}{5} \\ \frac{3}{5} & \frac{4}{5} & 0 \\ -\frac{16}{25} & -\frac{12}{25} & -\frac{3}{5} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[\beta_{ki}]^T[\beta_{kj}] = \begin{bmatrix} \frac{12}{25} & -\frac{9}{25} & -\frac{4}{5} \\ \frac{3}{5} & \frac{4}{5} & 0 \\ -\frac{16}{25} & -\frac{12}{25} & -\frac{3}{5} \end{bmatrix} \begin{bmatrix} \frac{12}{25} & \frac{3}{5} & -\frac{16}{25} \\ -\frac{9}{25} & \frac{4}{5} & \frac{12}{25} \\ -\frac{4}{5} & 0 & \frac{3}{5} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore \; [\beta_{ik}][\beta_{kj}]^T = [\beta_{ki}]^T[\beta_{kj}]$$

problem I-4

$$a_{ii} = a_{11} + a_{22} + a_{33} = 6$$

$$a_{ij}a_{ij} = a_{1j}a_{1j} + a_{2j}a_{2j} + a_{3j}a_{3j}$$

$$= (a_{11}a_{11} + a_{12}a_{12} + a_{13}a_{13}) = 24$$

$$+ (a_{21}a_{21} + a_{22}a_{22} + a_{23}a_{23})$$

$$+ (a_{31}a_{31} + a_{32}a_{32} + a_{33}a_{33})$$

c) $j=1, k=1$

$\rightarrow a_{i1} a_{i1} = a_{11}a_{11} + a_{21}a_{21} + a_{31}a_{31} = 2$

$j=1, k=2$

$\rightarrow a_{i1} a_{i2} = a_{11}a_{12} + a_{21}a_{22} + a_{31}a_{32}$

$= 1 + 2 + 0 = 3$

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Problem I-5

$\{\sigma'_{ij}\} = [Q_{ik}]^T \{\sigma_{kl}\} [Q_{lj}]$

$$[\sigma'_{ij}] = \begin{bmatrix} 1/2 & \frac{\sqrt{3}}{2} & 0 \\ -\frac{\sqrt{3}}{2} & 1/2 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 10 & 1 & 0 \\ 1 & 5 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1/2 & -\frac{\sqrt{3}}{2} & 0 \\ \sqrt{3}/2 & 1/2 & 0 \\ 0 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5+\frac{\sqrt{3}}{2} & \frac{1}{2}+\frac{5}{2}\sqrt{3} & \frac{\sqrt{3}}{2} \\ \frac{1}{2}-5\sqrt{3} & \frac{9}{2}-\frac{\sqrt{3}}{2} & \frac{5}{2} \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1/2 & -\frac{\sqrt{3}}{2} & 0 \\ \sqrt{3}/2 & \frac{1}{2} & 0 \\ 0 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{5}{2}+\frac{\sqrt{3}}{4}+\frac{\sqrt{3}}{4}+\frac{15}{4} & -\frac{5}{2}\sqrt{3}-\frac{3}{4}+\frac{1}{4}+\frac{5}{4}\sqrt{3}+\sqrt{3} & \frac{\sqrt{3}}{2} \\ \frac{1}{4}-\frac{5}{2}\sqrt{3}+\frac{9\sqrt{3}}{4}-\frac{3}{4} & -\frac{\sqrt{3}}{4}+\frac{15}{2}+\frac{9}{4}-\frac{\sqrt{3}}{4}+5 & \frac{5}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2}+2 & 1 \end{bmatrix}$$

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$$= \begin{bmatrix} \frac{25}{4}+\frac{\sqrt{3}}{2} & -\frac{1}{2}-\frac{\sqrt{3}}{4} & \frac{\sqrt{3}}{2} \\ -\frac{1}{2}-\frac{\sqrt{3}}{4} & \frac{59}{4}-\frac{\sqrt{3}}{2} & \frac{5}{2} \\ \frac{\sqrt{3}}{2} & \frac{5}{2} & 1 \end{bmatrix}$$

It has symmetry

Problem 2. 5

1st term : $\frac{\partial \underline{v}}{\partial t} = \frac{\partial v_i}{\partial t} \underline{e}_i$

2nd term : $\underline{v} \cdot \text{grad} = (v_i \underline{e}_i) \cdot (\frac{\partial}{\partial x_j} \underline{e}_j)$

$= v_i \frac{\partial}{\partial x_j} \underline{e}_i \cdot \underline{e}_j = v_i \frac{\partial}{\partial x_j} \delta_{ij} = v_i \frac{\partial}{\partial x_i}$

$(\underline{v} \cdot \text{grad}) \underline{v} = v_i \frac{\partial}{\partial x_i} (v_j \underline{e}_j) = v_i \frac{\partial v_j}{\partial x_i} \underline{e}_j$

$= v_j \frac{\partial v_i}{\partial x_j} \underline{e}_i$

3rd term $\underline{f} = f_i \underline{e}_i$

4th term : $\frac{1}{\rho}(\frac{\partial}{\partial x_i} \underline{e}_i) p = \frac{1}{\rho}\left(\frac{\partial p}{\partial x_i}\right) \underline{e}_i$

5th term . $\nu \Delta \underline{v} = \nu \frac{\partial^2}{\partial x_j \partial x_j} (v_i \underline{e}_i) = \nu \frac{\partial^2 v_i}{\partial x_j \partial x_j} \underline{e}_i$

$\therefore \frac{\partial v_i}{\partial t} + v_j \frac{\partial v_i}{\partial x_j} = f_i - \frac{1}{\rho} \frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 v_i}{\partial x_j \partial x_j}$

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