10/24 | CE131 Midterm 1 | Theresa Curtin

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Problem 1

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assuming that the equations given are ~~for~~

σ_0 state:

$$\Rightarrow \sigma_r\Big|_{\substack{r=a\\ \theta=\pi/2}} = \frac{\sigma_0}{2}\left[\left(1-\frac{a^2}{r^2}\right)+\left(1+\frac{3a^4}{r^4}-\frac{4a^2}{r^2}\right)\cos 2\theta\right] + \frac{-3\sigma_0}{2}\left[\left(1-\frac{a^2}{r^2}\right)+\left(1+\frac{3a^4}{r^4}-\frac{4a^2}{r^2}\right)\cos 2\theta\right]$$

$$= \frac{\sigma_0}{2}\left[0+(1+3-4)\cos\pi\right] - \frac{3\sigma_0}{2}\left[(1-1)+(1+3-4)\cos 2\pi\right]$$

$$\boxed{\sigma_r = 0}$$

$$\Rightarrow \sigma_\theta\Big|_{\substack{r=a\\ \theta=\pi/2}} = \frac{\sigma_0}{2}\left[\left(1+\frac{a^2}{r^2}\right)-\left(1+\frac{3a^4}{r^4}\right)\cos 2\theta\right] + \frac{-3\sigma_0}{2}\left[\left(1+\frac{a^2}{r^2}\right)-\left(1+\frac{3a^4}{r^4}\right)\cos 2\left(\theta+\frac{\pi}{2}\right)\right]$$

$$= \frac{\sigma_0}{2}\left[(1+1)-(1+3)\cos\pi\right] - \frac{3\sigma_0}{2}\left[(1+1)-(1+3)\cos 2\pi\right]$$

$$= \frac{\sigma_0}{2}\left[2+4\right] - \frac{3\sigma_0}{2}\left[2-4\right]$$

$$\boxed{\sigma_\theta = \frac{6\sigma_0}{2} + \frac{6\sigma_0}{2} = 6\sigma_0}$$

$$\Rightarrow \sigma_{r\theta}\Big|_{\substack{r=a\\ \theta=\frac{\pi}{2}}} = -\frac{\sigma_0}{2}\left(1-\frac{3a^4}{r^4}+\frac{2a^2}{r^2}\right)\sin 2\theta + \frac{-3\sigma_0}{2}\left(1-\frac{3a^4}{r^4}+\frac{2a^2}{r^2}\right)\sin 2\left(\theta+\frac{\pi}{2}\right)$$

$$= -\frac{\sigma_0}{2}(1-3+2)\sin\pi + \frac{3\sigma_0}{2}(1-3+2)\sin 2\pi$$

$$\boxed{\sigma_{r\theta} = 0}$$

Problem 2

a) $\nabla^4 \phi = \left(\frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4}\right)(ay^5 + bx^2y^3) = 0$

$0 = 0 + 0 + 0 + 24by + 120ay + 0$

$24b + 120a = 0$

$b = 5a$

b) $\sigma_{xx} = \frac{\partial^2 \phi}{\partial y^2} = 20ay^3 + 6bx^2y = 20ay^3 + 30ax^2y$

$\sigma_{yy} = \frac{\partial^2 \phi}{\partial x^2} = 2by^3 = 10ay^3$

$\sigma_{xy} = -\frac{\partial^2 \phi}{\partial x \partial y} = -6bxy^2 = 30axy^2$

Problem 3

1) $\sigma = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 3 & \sqrt{3} \\ 0 & \sqrt{3} & 0 \end{bmatrix}$

$$\det \begin{vmatrix} -\sigma & 1 & 0 \\ 1 & 3-\sigma & \sqrt{3} \\ 0 & \sqrt{3} & -\sigma \end{vmatrix}$$

$$= -\sigma(\sigma^2 - 3\sigma - 4) = 0$$

$$\sigma = -1,\ 0,\ 4$$
$$\quad \overset{\shortparallel}{\sigma_1} \quad \overset{\shortparallel}{\sigma_2} \quad \overset{\shortparallel}{\sigma_3}$$

$\underline{\sigma_1 = -1}$

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 4 & \sqrt{3} \\ 0 & \sqrt{3} & 1 \end{bmatrix} \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix} = 0 \qquad n_1 = -n_2 \qquad n_3 = -\sqrt{3} n_2 = \sqrt{3} n_1 \qquad n_1 = \begin{bmatrix} 1/\sqrt{5} \\ -1/\sqrt{5} \\ -\sqrt{3}/\sqrt{5} \end{bmatrix}$$

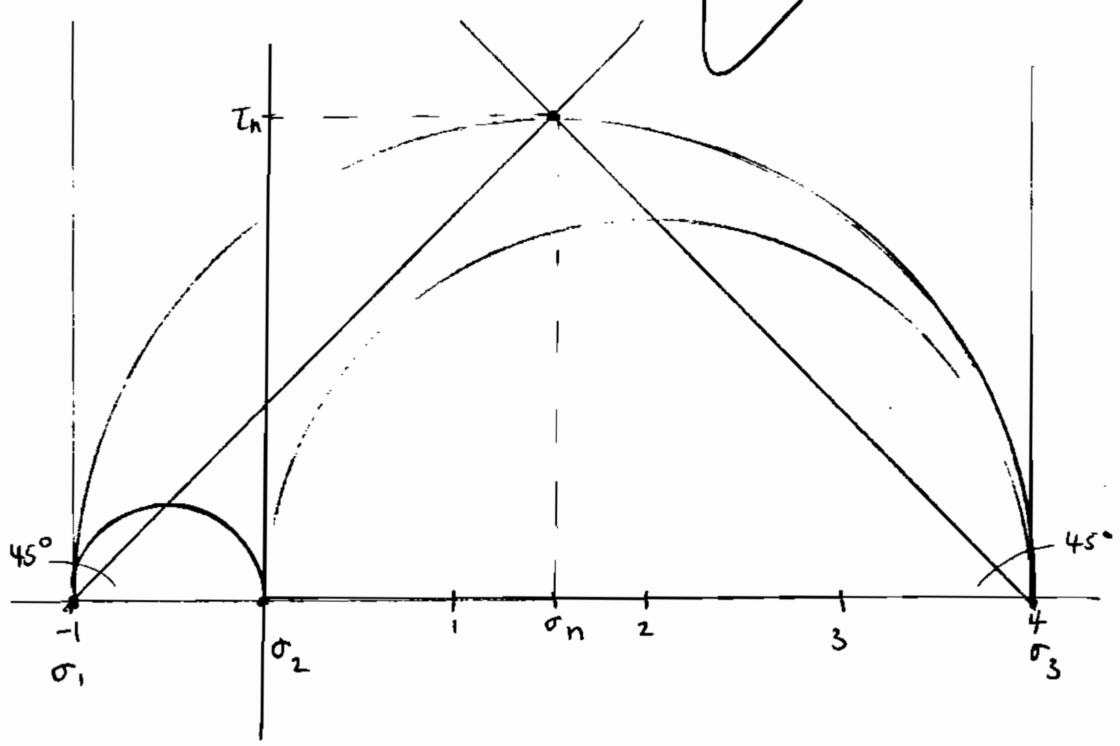
$\underline{\sigma_2 = 0}$

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 3 & \sqrt{3} \\ 0 & \sqrt{3} & 0 \end{bmatrix} \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix} = 0 \qquad n_2 = 0 \qquad n_1 = -\sqrt{3} n_3 \qquad n_2 = \begin{bmatrix} \sqrt{3}/2 \\ 0 \\ -1/2 \end{bmatrix}$$

$\underline{\sigma_3 = 4}$

$$\begin{bmatrix} -4 & 1 & 0 \\ 1 & -1 & \sqrt{3} \\ 0 & \sqrt{3} & -4 \end{bmatrix} \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix} = 0 \qquad n_1 = \tfrac{1}{4} n_2 \qquad n_2 = \tfrac{4}{\sqrt{3}} n_3 \qquad n_3 = \begin{bmatrix} 1/\sqrt{20} \\ 4/\sqrt{20} \\ \sqrt{3}/\sqrt{20} \end{bmatrix}$$

2)



$\sigma_n = 1.5$ MPa $\tau_n = 2.5$ MPa

Problem 4

a) $\sigma_{ij} = \lambda \epsilon_{kk} \delta_{ij} + 2\mu \epsilon_{ij}$

$2\mu\epsilon_{ij} = \sigma_{ij} - \lambda\epsilon_{kk}\delta_{ij}$

$\epsilon_{ij} = \frac{1}{2\mu}(\sigma_{ij} - \lambda\epsilon_{kk}\delta_{ij})$

b) $\sigma_{ii} = \lambda\epsilon_{kk}\delta_{ii} + 2\mu\epsilon_{ii} = 3\lambda\epsilon_{ii} + 2\mu\epsilon_{ii}$ $\boxed{\delta_{ii} = 1+1+1 = 3}$

$\sigma_{ii} = (3\lambda + 2\mu)\epsilon_{ii}$

c) $\epsilon_{ij} = \frac{1}{2\mu}(\sigma_{ij} - \lambda\epsilon_{kk}\delta_{ij})$

$= \frac{\sigma_{ij}}{2\mu} - \frac{1}{2\mu}\lambda\frac{\sigma_{kk}}{(3\lambda+2\mu)}\delta_{ij}$

$\epsilon_{ij} = \frac{\sigma_{ij}}{2\mu} - \frac{\lambda\sigma_{kk}\delta_{ij}}{2\mu(3\lambda+2\mu)}$

Problem 5

a) $\dfrac{\partial \sigma_{xx}}{\partial x} + \dfrac{\partial \sigma_{yx}}{\partial y} + \dfrac{\partial \sigma_{zx}}{\partial z} = 0$ ①

$2abx - 2abx + 0 \overset{\checkmark}{=} 0$

$\dfrac{\partial \sigma_{xy}}{\partial x} + \dfrac{\partial \sigma_{yy}}{\partial y} + \dfrac{\partial \sigma_{zy}}{\partial z} = 0$ ②

$-2aby + 2aby + 0 \overset{\checkmark}{=} 0$

$\dfrac{\partial \sigma_{xz}}{\partial x} + \dfrac{\partial \sigma_{yz}}{\partial y} + \dfrac{\partial \sigma_{zz}}{\partial z} = 0$ ③

$0 + 0 + 0 \overset{\checkmark}{=} 0$

yes, it does satisfy

b) This is not plane stress b/c $\sigma_{zz} \neq 0$.

For Plane strain:

$\epsilon_{zz} = \epsilon_{xz} = \epsilon_{yz} = 0$

$\epsilon_{xz} = \frac{1}{2G}\sigma_{xz} \overset{\checkmark}{=} 0$

$\epsilon_{yz} = \frac{1}{2G}\sigma_{yz} \overset{\checkmark}{=} 0$

$\epsilon_{zz} = \frac{1}{E}(-\nu\sigma_{xx} - \nu\sigma_{yy} + \sigma_{zz}) = 0$

$\epsilon_{zz} = \frac{1}{E}\left(-\nu(ay^2 + a\cancel{bx^2} - a\cancel{by^2}) - \nu(ax^2 + a\cancel{by^2} - a\cancel{bx^2}) + abx^2 + aby^2\right)$

$= \frac{1}{E}\left(-\nu(ax^2 + ay^2) + abx^2 + aby^2\right) \neq 0$

unless $b = \nu$

So, this is also not a plane strain problem.

for plane strain:

$\epsilon_{zz} = 0 = \frac{1}{E}(-\nu\sigma_{xx} - \nu\sigma_{yy} + \sigma_{zz})$

$0 = \frac{1}{E}\left(-\nu(ay^2 + a\cancel{bx^2} - a\cancel{by^2}) - \nu(ax^2 + a\cancel{by^2} - a\cancel{bx^2}) + abx^2 + aby^2\right)$

$0 = \frac{1}{E}\left(-\nu(ay^2 + ax^2) + abx^2 + aby^2\right)$

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Problem 5 cont'd

c) This is not plane strain, but if it were:

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)(\sigma_{xx}+\sigma_{yy})=0$$

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)(ay^2+ax^2)=0$$

$$2a+2a=4a\neq 0$$

The stress field does not satisfy.