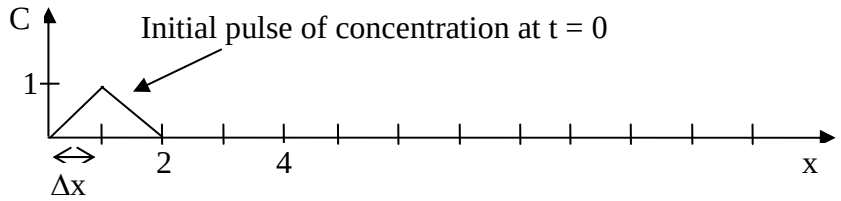
Comprehensive exam – CE200B Numerical modeling of environmental flows

Consider the advection equation for a scalar *C*:

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} = 0$$

At t = 0 a pulse of dye is injected into a one-dimensional channel as shown below. The boundary conditions are such that C(x=0, t) = 0. The velocity *u* is constant (*u* = 2 m/s).



a) Write an explicit first-order (in time and space) finite difference scheme to solve this equation. Choose parameters to make the solution as accurate as possible for the given grid spacing Δx = 1 m.
b) Carry out the first two time step iterations by hand for the first 6 grid points (x = 0 to 5 m). Give your results in table format, and then sketch and label the solutions on the axes above with the corresponding time in seconds.
c) If the velocity were to vary in space, what conditions would be required to guarantee stability of your scheme?