

Comprehensive exam – CE200B Numerical modeling of environmental flows

A river can be approximated by the long rectangle shown in the sketch. The flow is unidirectional and dispersion is significant only in the transverse direction. A chemical is discharged at the rate of Q at (x_0, y_0) . Initially the river contains no chemicals and clean water enters the river from upstream with a constant velocity u . The outflow boundary is open. Under these conditions, the governing equation for transport of the chemical reads

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} = D_y \frac{\partial^2 C}{\partial y^2} + \delta(x - x_0) \delta(y - y_0) Q$$

- Write an explicit finite difference scheme using leapfrog time advancement and central differencing in space to solve this equation. State the order of these approximations.
- Write the boundary conditions for each boundary in finite difference form and explain their meaning.
- Explain why special treatment is needed for the first time step and propose an appropriate scheme (without writing out the details).

