Comprehensive Examination in Dynamics

When needed, consider the spectral pseudo-acceleration response spectrum shown to the right in your solutions to the following problems.

1. Estimate the Peak Ground Acceleration (in %g) of the ground motion represented by the response spectrum shown.

2. What is the period $T_o$ where the spectrum transitions from the constant acceleration to the constant velocity range?

3. For an elastic single degree-of-freedom system, weighing 1000 kips, what is the minimum lateral stiffness of the system permitted such that it does not displace laterally more than 4 inches?

4. If the single degree-of-freedom system resulting from Part 3 has no viscous damping by itself, but a special damping device is added that is characterized as linear viscous damping ($F_d=Cv$) with a coefficient $C=1.25/T$ (expressed in k-sec/in.), what would the structure’s damping ratio be?

4. A colleague has handed you a sheet containing the dynamic properties of a two degree-of-freedom system, but you have managed to spill some coffee on the sheet and you can no longer read some of the numbers. You have no specific information on the magnitude and distribution of stiffness.

$$\begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$M = \frac{1000\text{kips}}{g} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix} \quad z = \begin{bmatrix} 1 \\ ? \end{bmatrix} \quad T = \begin{bmatrix} 1\text{sec.} \\ ? \end{bmatrix}$$

a. Based on the information provided, determine the modal coordinate for the second mode at the lower level.

b. How much of the structure’s total mass contributes to the base shear of the structure in the first mode?
Problem 1: (30 points)

An SDF system is excited by a sinusoidal force. At resonance the amplitude of displacement was measured to be 5 in. At an exciting frequency of one-tenth the natural frequency of the system, the displacement amplitude was measured to be 0.1 in. Estimate the damping ratio of the system.
Problem 2: (70 points)

The natural vibration frequencies and modes of vibration of the 2-story frame are:

\[ \omega_1 = 0.765 \sqrt{k/m} \quad \omega_2 = 1.848 \sqrt{k/m} \]

\[ \phi_1 = \begin{bmatrix} 1/\sqrt{2} \\ 1 \end{bmatrix} \quad \phi_2 = \begin{bmatrix} -1/\sqrt{2} \\ 1 \end{bmatrix} \]

where \( k = 24 \frac{EI}{h^3} \).

If the frame is excited by horizontal ground motion \( \ddot{u}_g(t) \), determine (a) the floor displacement response in terms of \( D_n(t) \), (b) the story shears in terms of \( A_n(t) \), and (c) the first-floor and base overturning moments in terms of \( A_n(t) \).
Problem 1

An SDF system is excited by a sinusoidal force. At resonance the amplitude of displacement was measured to be 2 in. At an exciting frequency of one-tenth the natural frequency of the system, the displacement amplitude was measured to be 0.2 in. Estimate the damping ratio of the system.
Problem 2

For the two shear frame excited by horizontal ground motion, determine the story shears in terms of $A_n(t)$, the pseudo-acceleration response of the $n$th-mode SDF system. Both stories have the same story stiffness $k$; the natural frequencies are $\omega_1 = 0.765 \sqrt{k/m}$, $\omega_2 = 1.848 \sqrt{k/m}$, and modes are $\varphi_1 = (1/\sqrt{2} , 1)^T$, $\varphi_2 = (-1/\sqrt{2} , 1)^T$. 

![Diagram of a two-story shear frame with rigid beams and elastics labeled EI, h, and 2h.]