SEMM Mechanics MS Comprehensive Exam Spring 2014

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1. Consider the following two-dimensional displacement field

$$\boldsymbol{u}(x,y) = ax^2y\boldsymbol{e}_x + by^3\boldsymbol{e}_y\,,$$

where a and b are scalar parameters with appropriate dimensions, x and y are the Cartesian coordinates, and $\boldsymbol{e}_x$ and $\boldsymbol{e}_y$ are the unit vectors in the coordinate directions. Compute the small strain tensor field for this state of deformation.

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2. Consider the following two-dimensional state of stress

$$\boldsymbol{\sigma} = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix} \text{ MPa}\,.$$

Find the principal stresses and the principal directions.

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3. Consider a one-dimensional Maxwell (fluid) with constitutive relation $\dot{\varepsilon} = \frac{1}{E}\dot{\sigma} + \frac{1}{\eta}\sigma$, where E is the stiffness and η the viscosity. Assume that the material is subjected to a step strain loading $\varepsilon(t) = \varepsilon_o H(t)$ and compute the stress response for the material. [Note: ε_o is a given scalar and $H(\cdot)$ is the Heaviside step function.]

University of California
Department of Civil and Environmental Engineering

Spring 2009
SEMM

Comprehensive Exam: Mechanics

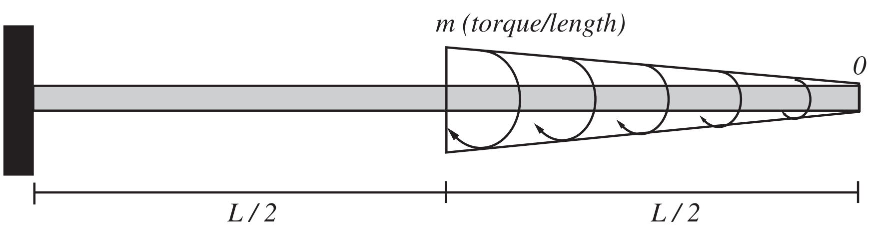
Name: ______________________________

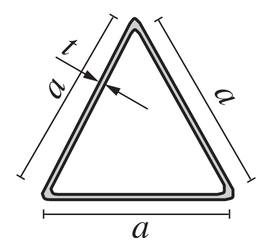
Problem #1

A shaft of length L and triangular tubular cross section as shown in the figure ($t \ll a$) is fixed at the left end while being subjected to a distributed torque as shown. The distributed torque varies linearly between the value m at the center and 0 at the free end.

1. Determine the angle of twist at the free end of the shaft if the material is linear elastic with shear modulus G.
2. If the material can only take a stress τ_y in shear before yielding, determine the maximum value m_y that can be applied before the shaft starts yielding.

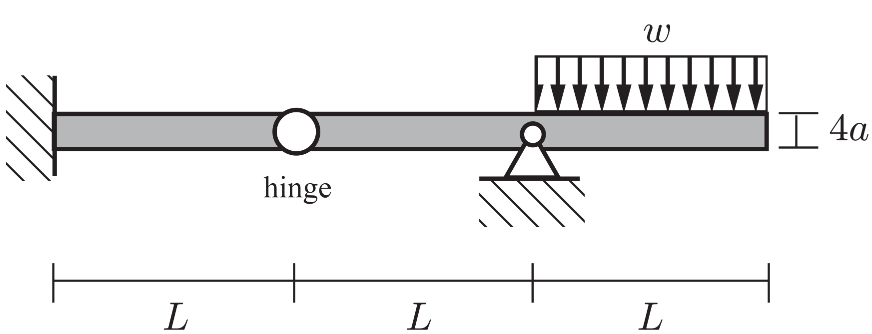
Remark: Express your answers in terms of L, a, t, m, G and τ_y, as required.

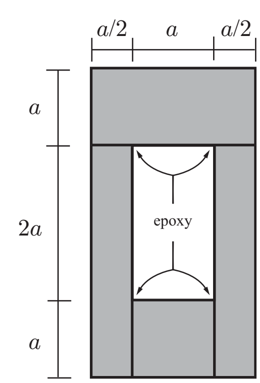




Problem #2

The beam shown in the figure is loaded by a uniform load w along the overhang. Its cross section is made by gluing together four planks with epoxy, as shown in the figure. Determine the minimum strength of the epoxy in terms of L, a and w, as needed.





University of California
Department of Civil and Environmental Engineering

Fall 2006
SEMM

Comprehensive Exam: Mechanics

Name: ____________________________________

1. Determine the deflected shape of an (Euler-Bernoulli) beam of span L and constant bending stiffness EI under its own weight (with constant weight/length ω) if both ends are clamped. Draw also the bending moment and shear force diagrams, indicating the characteristic values (including maximum and minimum values, and the reacting forces and moments at both ends).

2. Consider the state of plane stress $\sigma_{xx} = 9$ MPa, $\sigma_{yy} = 3$ MPa and $\tau_{xy} = -4$ MPa

   a. Draw the associated Mohr circles. Obtain the principal stresses and the principal directions (clearly define these directions with respect to the original Cartesian reference system, drawing a block oriented along these directions with the corresponding stresses).

   b. If the material is observed to yield when the maximum shear stress reaches the value $\tau_{max} = 6$ MPa, determine the additional stress σ_{zz}, in both tension and compression, that can be superposed in the perpendicular direction without yielding.

Student's Name (Please Print) ____________________

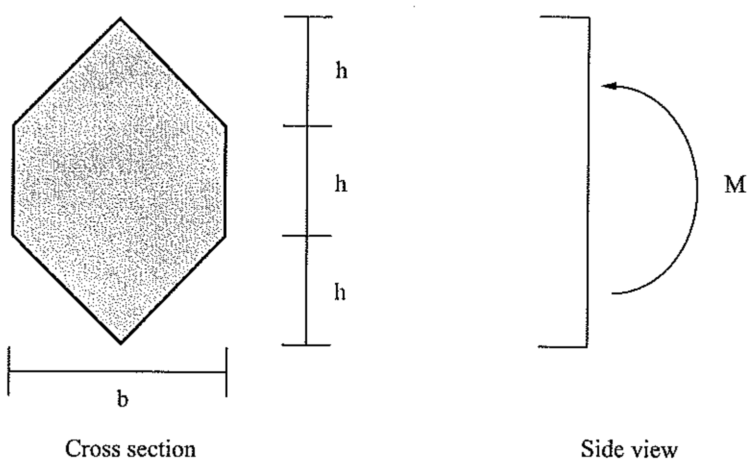
UNIVERSITY OF CALIFORNIA AT BERKELEY
Department of Civil and Environmental Engineering
SEMM

COMPREHENSIVE EXAM
Fall 2002
MECHANICS

Mechanics

Problem #1

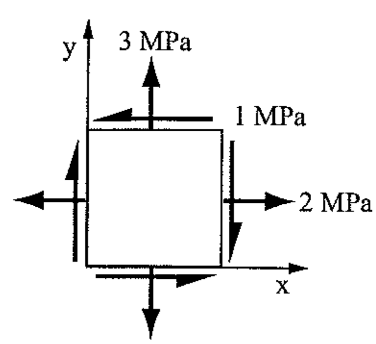
Determine the ratio M_{ult}/M_{yp} between the maximum elastic moment M_{yp} and the ultimate moment M_{ult} of an hexagonal cross section when loaded with a bending moment as shown in the figure. The material has the same yield limit σ_{yp} in tension and compression. Sketch the stresses under each of these moments.



Problem #2

Consider the state of stress depicted in the figure.

1. Draw the associated Mohr circle. Obtain the principal stresses, and sketch the principal directions (clearly define these directions with respect to the original Cartesian reference system, drawing a block along these directions with the corresponding stresses).
2. If the uniaxial yield limit of the material is $\sigma_{yp} = 4$ MPa, determine the range of the normal stress σ_{zz} that can be superposed to the above state of stress before the material yields according to Tresca.



University of California at Berkeley
Department of Civil and Environmental Engineering

Spring 2002
SEMM

Comprehensive Examination: Mechanics

Name ______________________________

Problem 1.

Consider a Bernoulli-Euler beam with rectangular cross section. The longitudinal stress on the beam cross section is given by the elastic flexure formula,

$$\sigma_x = -\frac{M(x)y}{I_z} \tag{1}$$

where I_z is the moment of inertia, which can be considered as a constant; $M(x)$ is the bending moment, which is a function of x; the vertical coordinate y measures the depth of the beam.

Use the equilibrium equation

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0 \tag{2}$$

to derive shear stress distribution along the depth of the beam

$$\tau_{xy} = \frac{V(x)}{2I_z}\left[\left(\frac{h}{2}\right)^2 - y^2\right] \tag{3}$$

Hints:

(1) the shear force $V(x)$ is given as

$$V(x) = \frac{\partial M(x)}{\partial x} \tag{4}$$

(2) The top and bottom surfaces of the beam is traction-free, i.e.

$$\tau_{yx}(\pm h/2) = 0 \tag{5}$$
$$\sigma_y(\pm h/2) = 0 \tag{6}$$

(50 points)

Problem 2.

Consider plane stress problem. For a fixed material point in a coordinate system XOY Cauchy stress tensor is given as

$$\sigma = \begin{pmatrix} \sigma_x & \tau_{xy} \\ \tau_{yx} & \sigma_y \end{pmatrix} \tag{7}$$

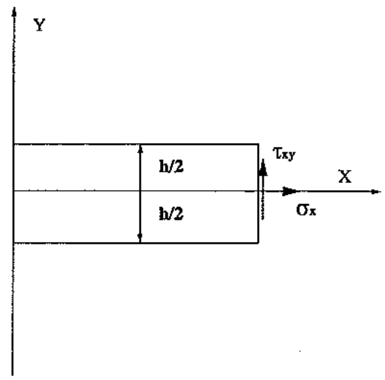


Figure 1: A Bernoulli-Euler beam

Consider a new coordinate system, $X'OY'$, which rotates an angle, θ, from the old coordinate system (see Fig 2.). The stress tensor has the components

$$\sigma' = \begin{pmatrix} \sigma'_x & \tau'_{xy} \\ \tau'_{yx} & \sigma'_y \end{pmatrix} \tag{8}$$

The relationship between old stress tensor components and new stress tensor components are

$$\begin{aligned} \sigma_{x'} &= \sigma_{x'}(\theta) = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta & (9) \\ \tau_{x'y'} &= \tau_{x'y'}(\theta) = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta & (10) \\ \sigma_{x'} &= \sigma_{x'}(\theta) = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta & (11) \end{aligned}$$

Find (1) The angle θ_p at which the normal stress component, $\sigma_{x'}(\theta_p)$, reaches to maximum, and the maximum value of $\sigma_{x'}(\theta)$, for $0 < \theta < 2\pi$. (2) The maximum shear stress, $\tau_{x'y'}(\theta_s)$, and the angle at which the maximum shear stress occurs. (50 points)

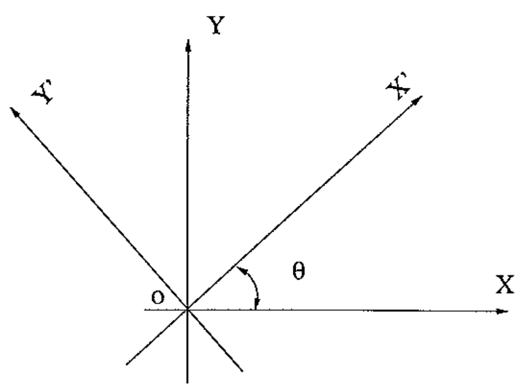


Figure 2: Transformation of coordinate

NAME: ______________________________

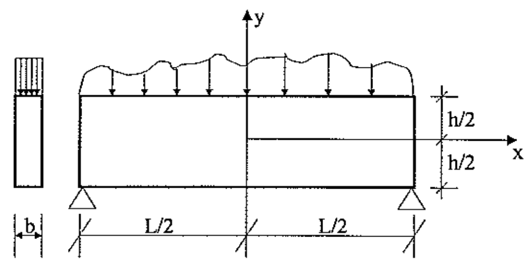
University of California at Berkeley — Department of Civil and Environmental Engineering

Structural Engineering, Mechanics and Materials — Spring, 2001

Comprehensive Examination
Mechanics

Problem # 1

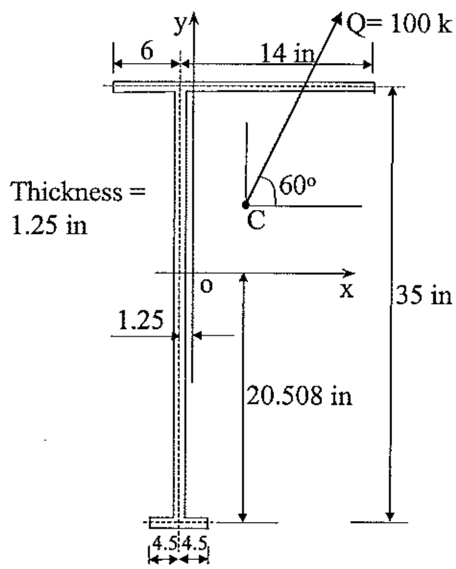
Consider the shown horizontal beam. According to the usual elementary theory of bending, the "fiber stress" is $\sigma_{xx} = -12\,M\,y/b\,h^3$, where M is the bending moment which is a function of x. Assume this value of σ_{xx}, and assume further that $\sigma_{zz} = \sigma_{zx} = \sigma_{zy} = 0$, that the body force is absent, that $\sigma_{xy} = 0$ at the top and bottom of the beam ($y = \pm h/2$), and that $\sigma_{yy} = 0$ at the bottom ($y = -h/2$). Derive σ_{xy} and σ_{yy} from the equations of equilibrium ($\sigma_{ij,j} = 0$, where $i = x$ or y and $j = x$ or y). Compare the results with those derived in elementary mechanics of materials.



Problem # 2

The shown cross section has its centroid at O (with the shown distances measured from the centerlines of the web and bottom flange) and is subjected to the shown shear force Q=100 k acting at an inclination of 60° to the horizontal. Determine the following:

a) the shear flow in the section (q)
b) the location of the shear center (i.e. coordinates of point C with respect to point O)



Hint: $q = \frac{Q_x'}{I_y'} S_y + \frac{Q_y'}{I_x'} S_x$, $Q_a'/I_b' = \begin{vmatrix} Q_a & Q_b \\ I_{ab} & I_a \end{vmatrix} \Big/ \begin{vmatrix} I_b & I_{ab} \\ I_{ab} & I_a \end{vmatrix}$, S_a is the first moment of area (on one side of the point of interest in the section) about a-axis, and Q_a is the shear force component along a-axis.