UNIVERSITY OF CALIFORNIA AT BERKELEY Department of Civil and Environmental Engineering
FALL SEMESTER 2015 Structural Engineering, Mechanics and Materials

Name ________________________________

Doctoral Preliminary Examination (Solid Mechanics)

Problem 1. (40 points)
Consider a single-connected domain Ω. The following boundary condition is prescribed,

$$\sigma_{ij} n_j = t_i = \Sigma_{ij} n_j, \ \ \forall \mathbf{x} \in \partial\Omega$$

where Σ_{ij} is a constant stress tensor.

Assume that the body force is zero, and the equilibrium equation inside the domain has the following form,

$$\sigma_{ij,j} = 0 \ .$$

Show that

$$\frac{1}{\Omega} \int_{\Omega} \sigma_{ij} \epsilon_{ij} d\Omega = \Sigma_{ij} < \epsilon_{ij} >$$

where

$$< \epsilon_{ij} > := \frac{1}{\Omega} \int_{\Omega} \epsilon_{ij} d\Omega \ .$$

Problem 2. (30 points)
Consider the following displacement field,

$$\begin{aligned} u_x &= \frac{1}{2} y^2 + \frac{1}{4} y^4 + xz \\ u_y &= \frac{1}{2} x^2 + \frac{1}{4} x^4 + yz \\ u_z &= xy \end{aligned}$$

1. Find the strain field ?
2. Find the rotation field ?
3. At the point $(1, 1, 0)$, there is a principal strain that has value -2.0. Find the other two principal strains.

Problem 3. (30 points)
Consider a stress tensor in plane stress state,

$$[\sigma_{ij}] = \begin{bmatrix} 3 & 1 \\ 1 & -1 \end{bmatrix}$$

Find suitable planes, i.e. $\mathbf{n} = (n_1, n_2)^T$, such that $\sigma_n = 0$.