Key Points:
- A scaling artifact confounds the interpretation of power law recession models
- A method is applied to remove the effects of the scaling artifact
- Artifact removal reveals new climatic patterns and, we argue, optimizes flow recession analysis

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a, b careful: The challenge of scale invariance for comparative analyses in power law models of the streamflow recession

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Abstract The falling limb of the hydrograph—the streamflow recession—is frequently well approximated by power law functions, in the form $dq/dt = -aq^b$, so that recessions are often characterized in terms of their power law parameters (a, b). The empirical determination and interpretation of the parameter a is typically biased by the presence of a ubiquitous mathematical artifact resulting from the scale-free properties of the power law function. This reduces the information available from recession parameter analysis and creates several heretofore unaddressed methodological "pitfalls." This letter outlines the artifact, demonstrates its genesis, and presents an empirical rescaling method to remove artifact effects from fitted recession parameters. The rescaling process reveals underlying climatic patterns obscured in the original data and, we suggest, could maximize the information content of fitted power laws.

1. Introduction

1.1. Modeling Streamflow Recessions With Power Laws

A broad spectrum of physically based and conceptual models of catchment hydrology use power law functions to represent the nonlinear reduction in discharge during streamflow recessions. Power law recession models describe either the relationship between discharge (q, in units of $[L/T]$ or $[L^3/T]$, where L represents a length scale and T a time scale) and its rate of change with time t [*Rupp and Selker*, 2006; *Botter et al.*, 2009; *Kirchner*, 2009], as:

$$\frac{dq}{dt} = -aq^b, \tag{1}$$

or between discharge and a metric of catchment storage volume (S, in units of $[L^3]$ or $[L]$), [*Brutsaert and Nieber*, 1977; *Wittenberg*, 1999; *Farmer et al.*, 2003; *Kirchner*, 2009], as

$$q = c\left(S - S_{ref}\right)^d, \tag{2}$$

where S_{ref} is the (generally) nonzero storage value at zero discharge.

The ability of power law functions to approximate observed nonlinearity in the hydrograph motivates their widespread adoption [*Amorocho*, 1963, 1967; *Kirchner*, 2009; *Wittenberg*, 1999]. Their use is often theoretically justified by the wide variety of solutions from hydraulic groundwater theory (which describes the discharge of an aquifer into a stream) that follow power law expressions of the form $dq/dt = -aq^b$ [*Rupp and Selker*, 2006; *Bogaart et al.*, 2013]. These models predict a particular value of the power law exponent b [*Rupp and Selker*, 2006; *Bogaart et al.*, 2013], while the coefficient a results from the interplay of several physical terms describing the aquifer geometry and its hydraulic properties. Power laws are also used as conceptual closure models for the catchment water balance, and in this context the exponent b is usually determined empirically by a best fit procedure [e.g., *Botter et al.*, 2009]. The multiplicative parameter a is also fitted and adopts a dimensionality that varies with the fitted exponent (i.e., the units of a are $T^{-1}\left(L^3/T\right)^{1-b}$, when q is a volumetric flow rate or $T^{-1}\left(L/T\right)^{1-b}$ when q is normalized by catchment area).

Power law behavior (at least approximately and over a finite range of scales) is a common phenomenon in natural, engineered, and social systems. Despite the ubiquity of power laws, their application is well known to raise a range of methodological pitfalls. For example, the common practice of log transforming a power law (generating a linear functional form) and then estimating its parameters via least squares regression creates a risk of bias in the fit. This arises because the least squares procedure places an equal weight on the linear

deviations from the line of best fit. Following back transformation into linear space, however, the magnitudes of these deviations diverge exponentially — biasing the fit toward small values of the model [e.g., *Miller*, 1984; *Pattyn and Van Huele*, 1998]. Similar problems arise when fitting power law distributions to data, due to biases introduced by binning and log transformation [*Goldstein et al.*, 2004; *Clauset et al.*, 2009]. These issues have been raised in several comprehensive reviews, which also demonstrate appropriate fitting and estimation techniques that avoid such biases [*Goldstein et al.*, 2004; *Clauset et al.*, 2009].

This letter addresses an additional mathematical property of power laws, their "scale-free" nature, which generates specific challenges for the analysis and interpretation of streamflow recessions. The methodological issues raised by the scale-free properties of power laws have not, however, received the same comprehensive analysis in the literature as the fitting and bias issues mentioned above. Although the work presented here is relevant to any power law model, the aim of this letter is to illustrate the consequences of the scale-free nature of the power law when used in hydrological settings — specifically recession analysis.

Firstly in section 1.2, we describe the scale-free nature of power laws and show how this property can generate mathematical artifacts that challenge interpretation of populations of fitted power law parameters. We then briefly address how these challenges affect two issues pertinent to catchment hydrology: (i) section 1.3 discusses the generation of tantalizing (but purely formal) relationships between model parameters, and (ii) section 1.4 outlines the potential for the mathematical artifact to obscure the information content of recession data and the drawbacks associated with current techniques for coping with power law scaling issues. To address these methodological challenges, section 2.1 outlines an empirical method to remove the scale-dependent artifact and shows that the resulting rescaled data reveals new, and potentially informative, temporal structure.

1.2. The Scale-Free Properties of Power Laws

The scale-free properties of power laws are most evident when the state variable (in this case the flow q) is scaled by a linear constant k, so that it adopts a new value $q = k\hat{q}$. Such rescaling is a basic data analysis operation — for example, it is required to change the units in which q is expressed or to normalize discharge by catchment area.

Following rescaling, if $\hat{q}$ is substituted into equation (1) and a is assumed independent of b, then a new power law relationship is obtained for the rescaled discharge:

$$\frac{\mathrm{d}\hat{q}}{\mathrm{d}t} = -ak^{b-1}\hat{q}^b, \tag{3}$$

Equation (3) has the same form as equation (1), and the exponent b is unchanged between the two equations. The equivalence in exponent is the reason that power laws are considered "scale-free" — the exponent of the power law is independent of a linear rescaling of the state variable. The difficulties are introduced, however, in the multiplicative parameter. If a simple power law were fit to equation (3), then this multiplicative parameter would be found to adopt a new value, $\hat{a}$, which differs from the multiplicative factor a at the original scale by a factor of k^{b-1} — that is, $\hat{a} = ak^{b-1}$.

The immediate consequence of this scale dependence is that the degree to which a fitted value of $\hat{a}$ reflects the scale at which flow is measured or reported (as embodied by the value of k^{b-1}), or to which it provides information about physical processes (as embedded in the scale-independent value of a), is unknown. This makes it challenging to fully identify and use the information contained in the power law parameters. Avoiding this issue, most hydrologic studies either examine variation in fitted values of b, which is independent of scale, or fix the value of b in order to examine *relative* variation in fitted values of $\hat{a}$. Arguably, neither of these approaches is ideal: either fitted values of $\hat{a}$ cannot be compared due to variation in b or the fitting procedure is biased by the constraint that b remain constant. In section 2.1, we present a rescaling approach applied to $\hat{q}$ in order to estimate the value a while avoiding the necessity of fixing b during the fitting process.

1.3. Formal Parameter Correlation

The relationship between the fitted power law coefficient and scale, $\hat{a} = ak^{b-1}$, means that the parameters $\hat{a}$ and b of the fitted power law display a correlation which can emerge, disappear, strengthen, and weaken with the scale of measurement. Explicitly noting that correlation occurs and is purely "formal" — that is, it does not arise from any hypothesized physical relationship but derives from the mathematical features of power laws — is important for recession analysis, since the emergence of similar correlations in several other fields

has historically created distraction and controversy. We have identified studies showing power law parameter correlations in the material fatigue literature [*Zilberstein*, 1992; *Cortie*, 1991], electrical engineering [*Shih et al.*, 1990], and in the fluid mechanics of blood circulation [*Hussain et al.*, 1999]. Several of these studies interpret the correlation as being indicative of physical or biological mechanisms—an interpretation that is at best confounded by and at worst solely attributable to the formal correlations. Correlation between power law parameters has been observed in recession curves [*Krueger et al.*, 2010; *McMillan et al.*, 2014] but is not widely discussed in the recession literature. Such correlations (in our own experience!) are highly compelling when identified in empirical data. They are not readily identified when considering tests to alleviate "spurious correlation" (that is, they do not conform to the traditional definition of a spurious correlation arising between two indices that have a common component [*Kronmal*, 1993]). Although power law correlation is not discussed in the recession literature, it has been documented between the parameters of sediment rating curves [*Syvitski et al.*, 2000; *Thomas*, 1988; *Mather and Johnson*, 2014]. These studies either made no attempt to explore the correlation [*Syvitski et al.*, 2000] or while treating it as an artifact and removing it in the case of *Mather and Johnson* [2014] did not investigate its origin [*Thomas*, 1988; *Mather and Johnson*, 2014].

A further motivation to ensure that hydrologists are familiar with power law parameter correlation lies in new theories that are currently being proposed to explain the origin of power law recessions in physical terms [*Harman et al.*, 2009; *Biswal and Marani*, 2010]. For example, *Harman et al.* [2009] show that power law recessions can result from catchment heterogeneity. If the recession from a catchment is conceptualized as the superposition of the outflow from a population of distinct, parallel hillslopes, each behaving as a linear reservoir, then the cumulative recession behavior can follow a power law. Adopting a different conceptualization of the main drivers of streamflow recession, *Biswal and Marani* [2010] hypothesize that the expansion and contraction of the wetted drainage network, the so-called "active drainage network," could generate power law recession dynamics. We note that in the original presentation of each of these theories, the recession exponent, b, should remain fixed. Logical extensions of the theory, however, could result in predictions of covariation in the recession parameters with catchment condition—through relaxation of the assumption that hillslope contributions to the watershed are temporally stationary, in the case of *Harman et al.* [2009], or under conditions of spatial heterogeneity relative to the static river network [e.g., *Biswal and Kumar*, 2012].

Without speculating on the validity, or formal derivation, of such predictions, we note that unambiguously describing the covariation of power law parameters in empirical data, and attributing it to physical rather than mathematical drivers, is nontrivial. Hydrological theories may well predict correlations between recession parameters, but observations of such correlations are likely to be a questionable basis for testing such theory.

1.4. Challenges in Model Parameter Selection and the Interpretation of Empirical Data

The primary challenge to comparative recession analysis and modeling posed by scale invariance lies in the relative magnitude of the terms a and k^{b-1} that compose any fitted $\hat{a}$. Without a technique to unambiguously separate these terms, or serendipitously selecting a scale such that $\hat{a} \approx a$, the information content of a fitted value of $\hat{a}$ will likely be obscured by the scaling term. The extent to which this negatively impacts the use of a power law recession relationship is largely dependent on specific applications. There remains, however, a clear need to identify methods that cope with scale dependence and correlation to maximize the quality of modeling applications and the value of comparative recession analyses.

1.4.1. Power Law Recessions and Model Closure

If the power law is used as a closure relationship in a hydrologic model, a single $(\hat{a}, b)$ pair must be chosen for model parametrization. This is typically accomplished in one of three ways:

1. *An effective pair is computed using a measure of centrality on the population of fitted $(\hat{a}, b)$ pairs*. In this case, the centrality measure (e.g., the mean) may poorly represent the population of parameters if the correlation is strongly nonlinear. In particular, if a strong correlation exists between $\hat{a}$ and b of the form $\hat{a} = ak^{b-1}$, then mean effective values for $\hat{a}$ and b ($\mathbb{E}[\hat{a}]$ and $\mathbb{E}[b]$) will not obey the correlation relationship. This may result in inexplicably poor recession curve fits when mean recession parameter values are used for modeling purposes.

2. *An effective pair is determined by minimizing some metric of model error*. Here a fitted $(\hat{a}, b)$ pair is chosen to minimize an error function across a population of recessions and may not have physical meaning.

3. *An effective pair is determined by defining a lower envelope to data points on a logarithmically scaled scatterplot of $-\mathrm{d}sq/\mathrm{d}t$ versus q*. This method avoids issues with parameter correlation because it does not require

Table 1. Potential Issues Associated With the Various Methods Used to Obtain Recession Parameter Estimates

Goal	Method	Outcome	Literature
Recession variability analysis	Examine variation in fitted b	Cannot examine variation in $\hat{a}$	*Stoelzle et al.* [2013] and *Tague and Grant* [2004]
	Fix b and fit $\hat{a}$	Additional fitting error potentially introduced could bias $\hat{a}$ fits	*Biswal and Marani* [2010] and *Biswal and Marani* [2014] *McMillan et al.* [2014]
Hydrologic modeling	Obtain $(\hat{a}, b)$ pair that minimizes error across a population of recessions.	Values should not be interpreted physically	*Müller et al.* [2014]
	Fit $(\hat{a}, b)$ pairs and use a measure of centrality to represent a population of recessions.	Potential issues with nonlinear averaging	*Botter et al.* [2009] and *Ye et al.* [2014]
	Define a lower envelope to data points on a log-log plot of dq/dt versus q	Can underestimate b for individual recessions, only theoretically valid for modeling base flow	*Brutsaert and Nieber* [1977]

fitting a population of recession parameter pairs; the recession exponent is fixed, either by fitting a single lower envelope to the data or by defining one a priori through theoretical considerations. However, the procedure explicitly assumes that the power law recession will be used to model catchment base flow [see, e.g., *Brutsaert and Nieber*, 1977]. Additionally, a number of authors have demonstrated that this lower envelope may significantly underestimate the value of the recession exponent [*Biswal and Marani*, 2010; *Shaw and Riha*, 2012; *Rupp et al.*, 2009].

Despite these challenges, the primary objective in hydrologic modeling is generally to minimize some measure of model error. If this goal is suitably met, then recession parameter interpretation and the precise values adopted by $\hat{a}$ and b may be of secondary importance. This is not the case, however, when power law recession models are used as a data analytic tool.

1.4.2. Power Law Models and Recession Analysis

When not being used as a closure model, the parameterized recession relationship is generally intended to provide a means for data exploration. In this case, fitted $(\hat{a}, b)$ pairs are analyzed to measure or determine the physical drivers of recession variability. This form of analysis presents another dilemma: either variations in b are examined [*Tague and Grant*, 2004; *Clark et al.*, 2009; *Shaw and Riha*, 2012], at the cost of injecting scale-induced variations into the fitted value of $\hat{a}$, or b is held fixed and the relative values of $\hat{a}$ can be compared, at the expense of introducing bias into the fit by forcing b to remain constant [*Szilagyi et al.*, 1998; *Biswal and Marani*, 2010; *Bart and Hope*, 2014; *Biswal and Marani*, 2014; *Mutzner et al.*, 2013; *McMillan et al.*, 2014]. Neither approach is ideal, as either the information that could be obtained from the $\hat{a}$ parameter is discarded (in order to examine variation in b) or the estimates of the $\hat{a}$ parameter are potentially biased because b is fixed (also removing the potential to examine variations in b). The additional fitting error due to this bias can be significant, as we demonstrate in the case study in section 3.

Table 1 summarizes the inherent challenges and limitations imposed by recession parameter scale dependence, along with representative studies that utilize the variety of available methods for recession analysis. These literature examples highlight that addressing scale invariance has the potential to further optimize, expand, or generalize even highly influential studies. In particular, the ideal approach would be to choose the flow scaling for which $\hat{a} = a$, allowing a physically meaningful estimate of a independent of the value of b.

2. Recession Parameter Decorrelation

2.1. Decorrelation Method

The most general route to remove formal parameter correlation involves selecting a number q_0 such that rescaling $\hat{q}$ as $\hat{q} \rightarrow \hat{q}/q_0$ eliminates correlation of the form $\hat{a} \propto k^{b-1}$ between the power law parameters. Such a technique was formally derived by *Bergner and Zouhar* [2000], who showed that correlation of this form is

minimized for a unique value of q_0, given by

$$q_0 = \exp\left(-\frac{\sum_{i=1}^{n}(b_i - \overline{b})(\log(\hat{a}_i) - \overline{\log(\hat{a})})}{\sum_{i=1}^{n}(b_i - \overline{b})^2}\right), \tag{4}$$

where $\overline{b}$ and $\overline{\log(\hat{a})}$ are the arithmetic means of a set of fitted recession exponents $\{b_1, b_2, \ldots, b_n\}$ and a set of log-transformed fitted recession scale parameters $\{\log(\hat{a}_1), \log(\hat{a}_2), \ldots, \log(\hat{a}_n)\}$, respectively. The value in the exponent of equation (4) is also equivalent to the negative of the regression slope of the graph of $\{\log(\hat{a}_1), \log(\hat{a}_2), \ldots, \log(\hat{a}_n)\}$ versus $\{b_1, b_2, \ldots, b_n\}$.

We note that the computed value q_0 will equal $1/k$ and that this rescaling will return $\hat{q}$ to the same magnitude of the original flow variable (q), which was introduced with units of $[L^3/T]$ or $[L/T]$. Strictly speaking, however, q_0 has units equal to those of $\hat{q}$ [*Bergner and Zouhar*, 2000]; thus, the transformed flow variable will be dimensionless, and the decorrelated recession scale parameter (which we will refer to as a) will have units of $[T^{-1}]$.

This method also assumes that a itself is not a function of b. If it were, the computed q_0 would still minimize correlation of the fitted recession exponents and log-transformed fitted recession scale parameters but would not necessarily equal $1/k$. Section 2.2 more thoroughly discusses the implications a potential relationship (beyond the artifactual correlation) between $\log(a)$ and b.

As far as we are aware, *Mather and Johnson* [2014] provide the only application of this method in the geosciences by removing correlation between the parameters of a turbidity rating curve. These authors demonstrate seasonal patterns in the turbidity rating curve scale parameter and also suggest that power law decorrelation could improve the physical interpretation of turbidity rating curve parameters.

2.2. Interpreting Parameter Decorrelation and the Rescaling Constant, q_0

Exploration of the decorrelation scaling q_0 suggests that it covaries with catchment properties, such as catchment area and mean flow. Across our study catchments, we find that q_0 ranges from about 4–33% of the mean (averaging around 12% of the mean) and from about 25–125% of the median (averaging around 60% of the median). However, we note that q_0 is not directly a catchment property but rather a property of the $(\hat{a}, b)$ point cloud to which the decorrelation process is applied. The specific value of q_0 is therefore dependent upon the selection of points in that cloud and thus on the technique used to select and to fit the power law recessions.

After finding a numeric value for q_0 using methods from section 2.1, a rescaling of the flow variable by q_0 ($\hat{q} \rightarrow \hat{q}/q_0$) will shift the population of fitted recession scale parameters $\{\hat{a}_1, \hat{a}_2, \ldots, \hat{a}_n\}$ so that all correlation of the form $\hat{a} \propto k^{b-1}$ is removed. As mentioned in section 2.1, if the correlation relationship takes exactly the form, $\hat{a} = ak^{b-1}$, the scaling constant q_0 is exactly equal to $1/k$. The approach thus removes the formal correlation (and the associated scaling effects) imposed by the power law scale invariance between the (a, b) pairs. If, however, a nonexponential form of correlation also exists between a and b, in addition to any induced by the scale-free nature of the power law—that is, if $\hat{a} = a(b) \cdot k^{b-1}$—then rescaling by q_0 will transform the function $a(b)$, and q_0 will not be equal to $1/k$ [*Bergner and Zouhar*, 2000]. As a consequence, the decorrelation procedure will fail to remove artifactual correlation due to the presence of nonexponential a-b correlation. This would continue to confound the physical interpretation of the transformed recession scale parameter. It should be noted, however, that the decorrelation procedure is effectively a rescaling of the flow variable and therefore does not introduce any more bias than would scaling flow by area, or equivalently, choosing a particular set of flow units. In the absence of a priori information about the functional form of such mechanistic correlation, its identification from a-b clouds will generally be problematic. Still, as shown in the simple example below, power law parameter decorrelation is a promising method for obtaining information about catchments from fitted recessions.

3. Case Study: Seasonal Recessions
3.1. Catchment Selection and Recession Analysis

To demonstrate the potential value of applying parameter decorrelation to empirical recession data, we analyzed streamflow recessions from 16 seasonally dry catchments in Northern California and Southern Oregon. In seasonally dry regions, the great majority of annual precipitation falls during a wet season (typically October through April in the Western U.S.), which is followed by an extended dry season.
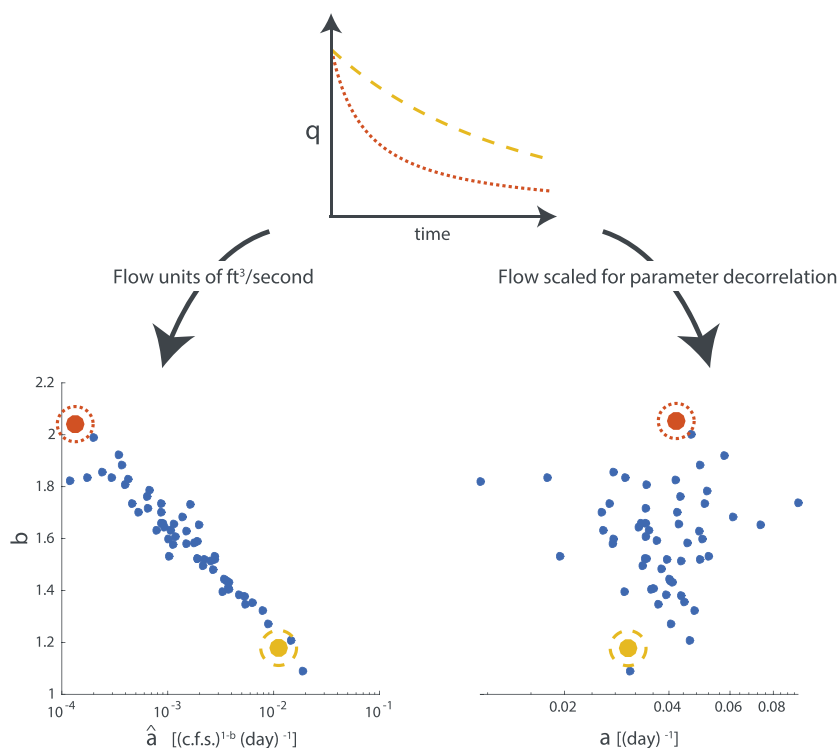
Figure 1. Illustration of the consequences of the parameter decorrelation procedure for 54 representative recessions extracted from Redwood Creek data. For $\hat{q}$ in units of c.f.s., there is a strong exponential correlation between $\hat{a}$ and b, as predicted by the rescaled power law recession relationship (equation (3)). Following parameter decorrelation, values of b remain constant, while values of a adjust such that the linear correlation between b and $\log a$ is zero. Two points along with the corresponding recession curves (orange and yellow) are tracked through the decorrelation procedure.

These locations, therefore, should reveal the effects of climatic variation on catchment wetness and the consequent properties of the streamflow recession. Details of these catchments are presented in the supporting information.

All streamflow peaks and subsequent recession periods $(d\hat{q}/dt < 0)$ greater than or equal to 4 days in length were first isolated from the streamflow time series. We computed $\hat{q}$ and $-d\hat{q}/dt$ using the procedure of *Brutsaert and Nieber* [1977] and found $\hat{a}$ and b by determining the line of best fit to the log-log plot of $-d\hat{q}/dt$ versus $\hat{q}$. To control the quality of fitted parameters, only recession fits with an $R^2 > 0.8$ were retained in the subsequent analyses. Although more precise forms of recession analysis are available, this particular method is widely used [*Biswal and Marani*, 2010; *Shaw and Riha*, 2012; *Mutzner et al.*, 2013; *Bart and Hope*, 2014] and its simplicity facilitates rapid analysis of many recessions. We confirmed that the fitting error imposed on $\hat{a}$ and b was negligible compared to the parameter shifts generated by the decorrelation procedure, which is the primary result we wish to demonstrate.

Parameter decorrelation was performed according to the method detailed in *Bergner and Zouhar* [2000]. Computed values of q_0 for each catchment are reported in the supporting information. Because q_0 is a property of the $(\hat{a}, b)$ point cloud, the values of q_0 quoted here are specific to the recession selection and fitting procedure used. Here our selection criteria included all well-defined peaks followed by at least 4 days of recession. This approach maximizes the range of flow conditions spanned by the analyzed recessions and ensures that the $(\hat{a}, b)$ point cloud used is as representative as possible of the full flow regime. All quantitative values of q_0 reported in the supporting information are specific to this peak selection method.

Following decorrelation, we computed the median value of b across all recession events for each basin. We then refit the set of recessions for each basin at the decorrelation scale, this time constraining b to equal its median value. Differences in the quality of this fit and fits with variable b provide an estimate of the additional fitting error accrued by fixing the recession exponent to a constant value.
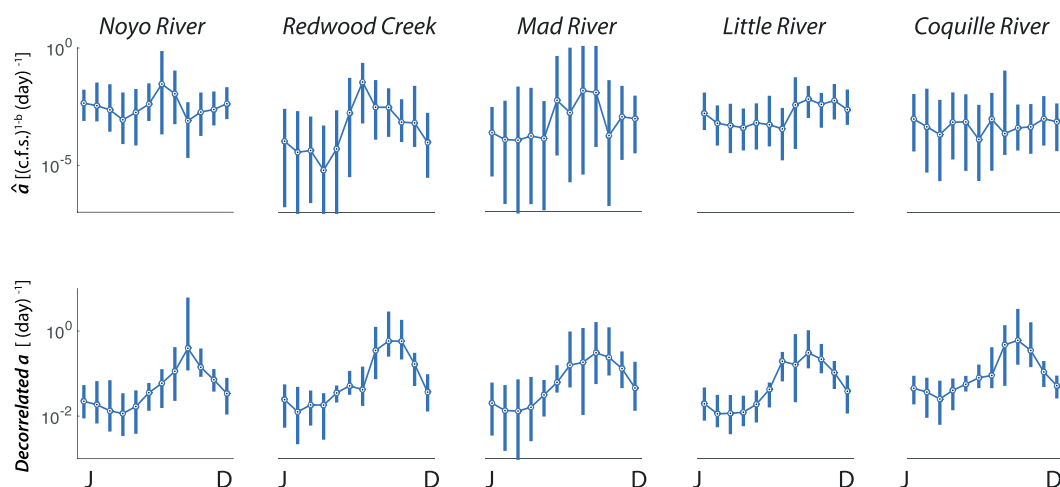
Figure 2. Examples of climatic patterns resulting from recession parameter decorrelation. Median monthly values (points) and 25th and 75th percentiles (boxes) for the recession scale parameter. (first row) The median monthly recession scale parameter is plotted for discharge in units of c.f.s. (second row) Clear seasonality in the recession scale parameter following the decorrelation procedure.

3.2. Case Study Results

Figure 1 illustrates the effects of the decorrelation process for a subset of recession parameter pairs value obtained at Redwood Creek. In Figure 1 (bottom left), $(\hat{a}, b)$ pairs with $\hat{q}$ in units of c.f.s. demonstrate a very clear correlation; following decorrelation (Figure 1, bottom right), recession exponent values remain the same, but the recession scale parameters have shifted significantly.

To date, most studies of recession parameter variability have considered the variation in a for a fixed value of b. To determine whether such estimates differ significantly from those produced by the decorrelation procedure, we first produce an $(\hat{a}, b)$ point cloud from the population of recessions associated with a given catchment. Applying the decorrelation rescaling procedure to this point cloud produces a population of correlation-free power law parameters (a_{free}, b). We then fix b at its median value and refit all recessions at the decorrelation scale to produce a population of estimates a_{fix}. We find that across all recessions in all catchments the median ratio $\rho = a_{\text{fix}}/a_{\text{free}}$ is 0.86, with a 25th percentile value of 0.42 and a 75th percentile value of 1.38. Assuming the value a_{free} at the decorrelation scale is the most accurate estimator of a, the spread in this ratio reveals potential for significant bias.

We then explored the time variation in the recession parameters. Prior to decorrelation, no coherent seasonal variation could be found in the fitted values of $\hat{a}$. Following decorrelation, however, a strong seasonal signal in the recession scale parameter can be seen in Figure 2, tracking the typical seasonality of rainfall. Similar seasonal patterns were observed in all 16 study catchments. We hypothesize that following decorrelation, the power law coefficient a may reveal information about the wetness of the catchment, exhibiting low values of a during the wet season and higher values during the dry season. Such recession seasonality has been observed previously, [*Shaw and Riha*, 2012], but it can also "disappear" (Figure 2) if b is not fixed, depending on scale. Generally, the arbitrary choice of scaling can mask (in some cases) or reveal (in others), the seasonally varying term, a. We hypothesize that identifying empirical correlates to recession variability — facilitated by the use of the decorrelation technique — could support the development and validation of improved hydrologic theory that links recession behavior to climatic and catchment wetness states.

4. Conclusion

Ubiquitous power law behavior in the recession of stream hydrographs leads to extensive use and analysis of power law parameters that characterize the drainage and drying of catchments. All such models will exhibit a scaling artifact that imposes a formal correlation between model parameters and which can obscure the scale-independent component of the power law, which is presumably most closely linked to the physical processes driving the recession behavior.

We apply a rescaling technique that eliminates the scale dependence of fitted power law terms under the assumption that there exists no mechanistic correlation between power law recession parameters. This technique allows both parameters of the power law to be estimated together without the imposition of artifactual bias. In the case that the power law recession parameters are expected to be correlated for mechanistic reasons, the rescaling technique will not add additional uncertainty and may still help to elucidate drivers of recession parameter variability. Whether it is possible to empirically separate mechanistic drivers of parameter correlation from mathematical artifacts that cause parameter correlation remains unclear.

As illustrated by the coherent seasonal variation of the power law multiplier *a* in seasonally dry watersheds from Northern California and Southern Oregon, this approach, although statistical rather than mechanistic in nature, may make clearer the relationships between catchment condition and the catchment recession.

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