A phenomenological model for the flow resistance over submerged vegetation

Alexandra G. Konings,1 Gabriel G. Katul,1,2 and Sally E. Thompson1

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[1] The bulk velocity $U_b$ in streams is conventionally estimated from Manning’s equation, but difficulties remain in parameterizing the roughness coefficient $n$ when the streambed is covered with vegetation. A two-layer velocity model is proposed to determine $n$ and $U_b$ for the submerged vegetation case. The modeled $n$ is derived as a function of flow and vegetation properties that can be inferred from remote sensing platforms, such as canopy height, leaf area density, and flow depth. The main novelty in the proposed formulation is that the shear stress is related to the mean velocity profile by considering both ejective and sweeping motions by dominant eddies. The proposed model is tested against a large data set from the literature and is shown to perform well, particularly for rigid vegetation. Poorer model performance for flexible vegetation can be partially attributed to the shape of the assumed mean velocity profile. The roughness coefficient $n$ is found to be robust to variations in the average spacing between canopy elements, allowing the model to be applied to heterogeneous canopies.


1. Introduction

[2] Methods to predict the bulk or area-averaged mean velocity $U_b$ in streams commonly employ Manning’s equation, which links $U_b$ to channel slope $S$ and hydraulic radius $R$ through a single roughness coefficient $n$. In practice, values of $n$ for a specific surface cover are selected using look-up tables [Chow, 1959]. Look-up table methods can be problematic for determining $U_b$ in vegetated channels because vegetation characteristics substantially influence the mean velocity profile $\bar{u}(z)$, where $z$ is the height from the channel bed and the overbar denotes Reynolds averaging. The vegetation structural properties on which $\bar{u}(z)$ depends vary in time and are not always known. However, remote sensing technologies do permit widespread characterization of vegetation attributes and flow characteristics. Interferometric synthetic aperture radar (InSAR) measurements allow for determination of variations in water surface elevation [Alsdorf et al., 2007]. The future Surface Water and Ocean Topography (SWOT) satellite mission (recently approved as a NASA Decadal Survey Mission) will use InSAR and other instruments to provide estimates of water surface elevations with an accuracy on the scale of centimeters [Durand et al., 2010]. Lidar methods can be used to characterize vegetation height ($h_v$) and leaf area density ($a$) for use as inputs to operational flood routing models [Antonarakis et al., 2010; Forzieri et al., 2010].

[3] Clearly, theoretical models to estimate $U_b$ on the basis of these vegetation and water elevation properties can be decisive in advancing flood routing predictions. Although several such models have been proposed [e.g., Wu et al., 1999; Musleh and Cruise, 2006; Wilson, 2007; Yang and Choi, 2010], these models have not generally taken advantage of what is known about the characteristics of the turbulent eddies that dominate momentum transport in vegetated systems. A model for $U_b$ and $n$ in channels with submerged vegetation is proposed here by extending the recent theoretical treatment of Gioia and Bombardelli [2002]. In this paper, a phenomenological approach is used to describe the momentum transfer at the canopy top on the basis of characterizing the dominant eddies, thereby closing the channel-wide momentum budget. In doing so, the effects of ejections and sweeps on momentum transfer associated with the eddies are simultaneously accounted for. Ejections occur when $u' < 0$ and $w' > 0$ and sweeps occur when $u' > 0$ and $w' < 0$ [Robinson, 1981], where $u = \bar{u} + u'$ and $w = \bar{w} + w'$ are the instantaneous streamwise and vertical velocities and primed quantities are excursions from the Reynolds-averaged quantities (indicated by overbar throughout).

[4] The proposed model is tested against (1) a large data set of measured values of $n$ and $U_b$ from the literature and (2) an existing two-layer model of flow in vegetation channels with regularly spaced rod elements [Huthoff et al., 2007]. The principal differences between the model proposed here and that of Huthoff et al. are that the proposed model accounts for the momentum contributions generated by ejections and explicitly includes constraints imposed on the eddy sizes on the basis of their ability to penetrate the canopy, both features excluded in the model of Huthoff et al. [2007].
2. Model Description

In section 2 the model for \( n \) and \( U_b \) is derived for the submerged vegetation case. The phenomenological approach introduced by Gioia and Bombardelli [2002] for cases where the channel roughness height is small (referred to as the rough boundary layer formulation) is first summarized to illustrate the origins of the model structure. A model aquatic canopy is then introduced, and its properties are used to adapt the rough boundary layer formulation to a channel covered with vegetation. The entire derivation for the vegetated channel model is shown.

2.1. The Rough Boundary Layer Formulation

Gioia and Bombardelli [2002] used a phenomenological approach to derive the turbulent shear stress \( \tau \) at the channel bed when the channel roughness height is small relative to \( h_n \). Gioia and Chakraborty [2006] and Gioia et al. [2010] also used a similar phenomenological model to quantify how variations in friction factor and mean velocity profile are impacted by variations in Reynolds number in pipe flow. In this approach, the roughness elements were assumed to have a length scale \( h_0 \). The \( \tau \) was assumed to be dominated by eddies that straddle the coves between successive roughness elements at the wall surface. The projection of this eddy morphology is a sphere whose radius \( h_0 \) centered on the plane tangent to the roughness elements and parallel to the bottom. The dominant eddy shape and channel system are illustrated in Figure 1. The \( \tau \) from this eddy depends on the velocities tangent \( (v_t) \) and normal \( (v_n) \) to the surface,

\[
\tau \sim \rho_w |v_t v_n|, \quad (1)
\]

where \( \rho_w \) is the density of water. The \( v_t \) depends on the difference between the velocity at the top of the eddy, which scales with \( U_b \) (in a single-layer model) and the velocity at the bottom of the eddy, which is assumed to be negligible relative to \( U_b \) because of the no-slip boundary condition. Thus, \( v_t \sim (U_b - 0) \). The \( v_n \) is the characteristic velocity bounding the eddy of size \( h_0 \) and normal to the surface. The magnitude of this velocity is characterized by the structure function \( v_n(h_0) = \left( \Delta w(x + h_0) - w(x) \right)^{1/2} \), such that \( v_n = \left( \Delta w(h_0) \right)^{1/2} \). For an eddy of radius \( h_0 \), Gioia and Bombardelli [2002] used the Kolmogorov four-fifths scaling law for the inertial subrange to estimate \( v_n(h_0) \), which is given as \[ \text{e.g., Kolmogorov, 1941; Tennekes and Lumley, 1972; Frisch, 1995} \]

\[
v_n = (\Delta w)^{1/2} = \frac{4}{5} \epsilon h_0, \quad (2)
\]

where \( \epsilon \) is the rate of dissipation of turbulent kinetic energy (TKE). This four-fifths scaling law is exact for locally homogeneous and isotropic turbulence, which may not characterize the dominant eddy here. If the Kolmogorov four-fifths scaling law is assumed to hold without any further modifications arising from TKE production or viscous dissipation, then equation (2) holds even for eddies extending throughout \( h_n \). Such eddies have a radius commensurate with the hydraulic radius \( R \) (equal to \( h_0 B \) for rectangular channels, where \( B \) is the channel width). The structure function of those eddies that scale with \( R \) at \( z = h_0 \) can be written as

\[
\langle \Delta w(R) \rangle = 2 \sigma^2 (1 - \rho(R)), \quad (3)
\]

where \( \sigma \) is the variance of the turbulent velocity, and \( \rho \) is its autocorrelation function. If the integral length scale of the turbulent velocity is smaller than \( h_n \), then \( \rho(R) \approx 0 \). As

![Figure 1. Schematic representation of the canopy and dominant eddy configuration. (left) The dominant eddies for the rough boundary layer formulation, which straddle the space between the roughness elements of radius \( h_0 \). The tangential velocity scale \( v_t \) associated with these eddies is \( U_b \) since there is no finite velocity at the bottom of the eddy. (right) A model for dominant eddies shown for aquatic vegetation. In this case, both ejections (associated with \( U = U_c \), the velocity in the canopy) and sweeps (associated with \( U = U_c + \Delta U \), the velocity above the canopy) contribute a finite velocity to \( v_t \).](image-url)
a result, \((\Delta w(R))^2 \sim 2\sigma^2\), a key assumption of Gioia and Bombardelli [2002]. Since the variance of the velocity scales as \(U_b^4\), \((\Delta w(R))^2 \sim U_b^2\). From Kolmogorov’s theory, then
\[
(\Delta w(R))^2 \sim U_b^0 \sim \epsilon^{2/3} R^{1/3},
\]
(4)
and for the third-order structure function,
\[
(\Delta w(R))^3 \sim U_b^1 \sim \epsilon^{5/3} R^{1/3}
\sim U_b^0 \sim \epsilon R.
\]
(5)
Combining equations (2) and (5) allows the estimation of \(v_n\) and \(\frac{v}{\rho_n}\) as
\[
v_n \sim U_b \left(\frac{h_0}{R}\right)^{1/3}. \tag{6}\]
Using equation (1),
\[
\frac{\tau}{\rho_n} \sim U_b U_{b0} \left(\frac{h_0}{R}\right)^{1/3}. \tag{7}\]
Assuming a uniform \(h_0\) and negligible wind shear stress at the channel surface, applying a force balance results in \(\tau/\rho_n = gRS\), where \(g\) is the gravitational acceleration. Combining this with equation (7) results in
\[
U_b \sim \left(\frac{h_0}{R}\right)^{-1/6} \sqrt{gSR}, \tag{8}\]
which recovers Manning’s equation if \(n^{-1} \sim h_0^{-1/6} \sqrt{g}\). The exponent \(1/6\) for \(h_0\) is the typical “Strickler” scaling, and the \(g^{-1/2}\) scaling has been derived or used by a number of studies [Chow, 1959; Carr, 1979; Komar, 1979; Katul et al., 2002].

### 2.2. The Submerged Canopy Formulation

[7] The approach above can be extended to determine \(U_b\) of a submerged aquatic canopy, also illustrated in Figure 1. The canopy consists of cylindrical elements of diameter \(d\) and height \(h_c\). The cylindrical canopy elements have a leaf area density \(a\), defined as the canopy frontal area per volume. Its mean value can be determined from an “equivalent” leaf area index (LAI) using \(a = \text{LAI}/h_c\). The density of the canopy elements in the channel bed, defined as the number of stems per unit area, is \(n_d\), such that the average spacing between canopy elements is \(S_c = n_d - d\). The drag exerted by the canopy is represented by a constant drag coefficient \(C_d\). As before, the flow in the channel has a depth \(h_w\), with \(h_w > h_c\). The submergence depth of the vegetation can be defined by the dimensionless height \(\alpha = h_c/h_w < 1\). A two-layer velocity model is now assumed on the basis of the fact that the average velocity in the canopy is finite but substantially lower than the average velocity above the canopy for dense canopies [e.g., Nepf and Vivoni, 2000].

The velocity is assumed to have a constant average value within the canopy \(U_s\) and a constant average value above it, with a discontinuity of size \(\Delta U\) occurring at the canopy top. The flow velocity above the canopy is therefore \(U_c + \Delta U\). Assuming \(\alpha\) is known, the \(U_b\) is given by,
\[
U_b = U_c + (1 - \alpha)\Delta U. \tag{9}\]

[8] As before, the shear stress at the top of the canopy \((\tau_c)\) is assumed to be dominated by eddies of a single optimal size \(s\) most efficient at transferring momentum. Such a dominant eddy is centered at the top of the canopy, as illustrated in Figure 1. As with the derivation of \(\tau\), \(\tau_c\) depends on \(v_n\) and \(v_s\). The \(v_s\) again depends on the difference between the velocity at the top of the eddy, now \(U_c + \Delta U\), and the velocity at the bottom of the eddy, now \(U_c\). Therefore, \(v_s \sim \Delta U\). As before, \(v_n\) is the characteristic velocity of eddies of size \(s\) \((u_s)\) and the Kolmogorov four-fifths law can be used so that
\[
u_s = U_b \left(\frac{s}{h_w}\right)^{1/3}. \tag{10}\]

[9] The use of the four-fifths law assumes that the Kolmogorov model of turbulence is an accurate description of the flow at the canopy top. Some evidence exists to support this assumption. Figure 2 presents an idealized version of vertical velocity energy spectra measured by Poggi et al. [2004a, Figure 5b] in a flume with submerged steel rods acting as canopy elements. The spectrum at a height near the canopy top \((z/h_c = 1.1)\) is close to the expected Kolmogorov spectrum in terms of scaling laws. Inside the canopy, the spectrum is complicated by numerous factors including wake production at Strouhal frequencies commensurate with the generation of Von Karman streets, shear production at larger scales, and the lack of Kolmogorov scaling often used in delineating the inertial subrange.

[10] Equation (10) also assumes that the \(h_w\) length scale is within the inertial subrange at \(z = h_c\). The canopy top spectrum in Figure 2 shows that while the inertial subrange extends to \(h_c\) at \(z/h = 1\), it does not extend to \(h_w\). However, the spectrum at \(z/h = 1\) resembles one with conventional descriptions of the modulations to the K41 spectrum by production and dissipation. Appendix A explores the use of a canonical spectrum with these modulations (that of Gioia et al. [2010], which resembles the spectrum at \(z/h \sim 1.9\)). Accounting for the energetic and dissipative regimes only has a minor effect on the predicted \(n_s\), suggesting that the use of Kolmogorov’s four-fifths law in describing \(u_s\) is a reasonable assumption up to scales commensurate with \(h_w\), at least at the canopy top. As shown in Figure 2, the use of the four-fifths law is not accurate for describing flow deeper in the canopy; hence using this approach is questionable when describing turbulent stresses at some arbitrary level inside the canopy.

[11] To evaluate \(u_s\) in equation (10), the size of the eddy dominating the shear stress must be known. The shear stress is dominated by the largest possible eddy centered at the top of the canopy. Both horizontal and vertical length scales can limit the eddy size. Vertically, the eddy radius \(s\) is constrained by the depth of water above the canopy.
Figure 2. Idealized version of the energy spectra measured at different heights in Poggi et al. [2004a]. The spectra are premultiplied by the wave number, such that a slope of \(-2/3\) corresponds to the slope of \(-5/3\) (= \(-2/3 - 1\)) in the usual Kolmogorov spectrum. The black solid line is the spectrum right above the canopy top, at \(z/h = 1.1\), while the red dashed line represents the spectrum deep within the canopy, at \(z/h < 1\). The blue dash-dotted line is the spectrum at a height well above the canopy top. The green vertical lines correspond to the frequencies associated with the water depth \(h_w\) and the canopy height \(h_c\) in the experiments of Poggi et al. [2004a], respectively.

The limiting vertical length scale is given by \(h_w(1 - \alpha)\) and by the depth to which the eddies can effectively penetrate the canopy. The penetration depth is conventionally defined as the depth at which the Reynolds’ stress is 10% of the top of the canopy value [Nepf and Vivoni, 2000]. Across a wide range of experiments, Nepf et al. [2004] found that the penetration depth \(\delta\) was given by

\[
\delta = \begin{cases} 0.21 \frac{C_{da}}{C_{u}}, & (C_{da}h_c)^{-1} \leq 4, \\ 0.85h_c, & (C_{da}h_c)^{-1} > 4. \end{cases}
\]

(11)

The limiting horizontal length scale is given by \(r = \min(h_w - h_c)\). The limiting horizontal length scale is given by \(B/2\). Since the canopy eddies do not continuously span the channel laterally, not all momentum is necessarily absorbed over the distance between canopy elements \(S_c\). The \(B/2\) is therefore expected to be a more appropriate horizontal length scale than \(S_c\). In the derivation here, relatively wide channels are assumed so that \(h_w < B\). As a result, \(r < B\) and \(s = r\). This eddy length scale is supported by the measurements of Ghisalberti [2010].

The phenomenological description of \(\tau_c\) can be used to close the momentum budget by employing a force balance on the layer of flow above the canopy. Assuming the flow is steady and uniform and that dispersive fluxes are small enough to be neglected [Poggi et al., 2004b], the force of gravity on the water above the vegetation canopy is balanced by the shear stress produced at the top of the canopy so that

\[
\rho_a g(h_w - h_c)S = \tau_c + \tau_w,
\]

(12)

where \(\tau_w\) is the drag on the channel walls above \(h_c\), which is assumed to be small relative to the drag at the top of the canopy. Evaluating the shear stress in equation (12) and simplifying gives

\[
g(h_w - h_c)S = K_c \frac{1}{h_w^{1/3}} U_h \Delta U,
\]

(13)

where \(K_c\) is a constant of proportionality.

[13] Applying a force balance to the entire channel (using the same assumptions as in equation (12)) provides an equation for \(U_c\) given as

\[
\rho_a g h_c S = \frac{1}{2} C_{da} h_c U_c^2 + \tau_b(0) + \tau_w(0),
\]

(14)

where the right-hand side is the drag force created by the canopy elements and the drag at the canopy bed \(\tau_b\) and on the channel walls \(\tau_w\). The \(\tau_b(0)\) and \(\tau_w\) are assumed here to be negligible relative to the canopy drag. With this simplification,

\[
U_c = \sqrt{\frac{2g S}{\alpha C_{da}}} = \sqrt{\frac{2g S l_c}{\alpha}}
\]

(15)

The quantity \(l_c = (C_{da})^{-1}\) is known as the adjustment length scale and is used in canopy turbulence studies to parameterize the loss of turbulent kinetic energy from advecting eddies due to their dissipation by the canopy drag elements [Finnigan, 2000; Belcher et al., 2003].

[14] Equations (9), (13), and (14) form a system of three equations with three unknowns, \(U_b\), \(U_c\), and \(\Delta U\). Combining the three equations leads to a single quadratic equation for \(U_b\),

\[
U_b^2 - \sqrt{2g S l_c} U_b - (1 - \alpha) g(h_w - h_c)S \left(\frac{h_w}{r}\right)^{1/3} = 0.
\]

(16)

Substituting Manning’s equation provides a single quadratic equation for \(n\):

\[
(1 - \alpha) g(h_w - h_c) \left(\frac{h_w}{r}\right)^{1/3} \left(\frac{h_w}{r}\right)^{1/2} + \sqrt{\frac{2g l_c}{\alpha}} - R^{2/3} = 0.
\]

(17)

Equation (17) has only a single positive root. Unlike \(U_b\), \(n\) is independent of \(S_c\), as expected. Note that for a vegetated channel, the Strickler scaling is not recovered when \(h_c\) goes to 0 since the bed stress \(\tau_b\) is assumed zero. If \(\tau_b\) is finite and accounted for, the Strickler scaling will be recovered for the \(h_c\) of the channel bed via the conventional arguments of Gioia and Bombardelli [2002].

3. Data Sets

[15] The model was tested using a data set consisting of a large range of laboratory measurements of flows with
submerged vegetation from the literature. It includes the data sets listed by Poggi et al. [2009] and Cheng [2011]. One hundred sixty-eight experiments in the data set examine rigid canopy elements [Fenzl, 1962; Dunn et al., 1996; Stone, 1997; Meijer and van Velzen, 1999; Lopez and Garcia, 2001; Ghisalberti and Nepf, 2004; Poggi et al., 2004a; Murphy et al., 2007; Liu et al., 2008; Nezu and Sanjou, 2008; Yang and Choi, 2009; Cheng, 2011], and 67 experiments considered flexible vegetation [Dunn et al., 1996; Carolla et al., 2002; Jarvela, 2005; Ciraolo and Ferreri, 2007; Kubrak et al., 2008; Yang and Choi, 2009]. When $C_{ghc}$ < 0.2, eddies may penetrate to the channel bed [Nepf et al., 2007], which would invalidate the assumption of a nonnegligible bed stress in equation (14). Because the model is only valid when $C_{ghc}$ > 0.2, only experiments that meet this condition are included.

[16] The drag coefficient was not measured in all of the studies used. Wherever an estimate of $C_d$ was used in the original study, that value was used here as well. When $C_d$ is unknown, $C_d = 1.13$ was used. This was the mean value measured experimentally by Dunn et al. [1996], matching analytical estimates of Li and Shen [1973]. This is also the value used by Lopez and Garcia [2001].

[17] Flexible vegetation elements can deflect, changing the geometry of the problem. Therefore, the rigid and flexible vegetation cases are considered separately. For flexible vegetation, $h_c$ is assumed to be the reported deflected canopy height. The experiments cover a wide range of $n$ (0.018–0.139 for rigid vegetation and 0.011–0.130 for flexible vegetation) that approximately spans the entire range of $n$ values typically assumed for natural channels [Chow, 1959]. The data set also spans a range of values for the Reynolds number, $\alpha$, $S$, and $R$. A table with parameter values for each of the data points is included as auxiliary material. A best fit value is found separately for the scaling parameter $K_r$ for rigid and flexible canopies.

4. Results

[18] Huthoff et al. [2007] was the first to apply the methodology of Gioia and Bombardelli [2002] to derive a two-layer velocity model for vegetated channels, using an approach similar to that employed here. Two crucial differences exist between the model of Huthoff et al. [2007] and the one proposed here: (1) Huthoff et al. argued that velocity fluctuations tangent to the streamflow are proportional to the average velocity above the canopy rather than to the difference in velocity above and below the canopy top, as argued here, and (2) the length scale used by Huthoff et al. is proportional to the horizontal spacing between regularly distributed canopy elements, rather than depending on the penetration depth or $h_c - h$, as argued here. Because the conceptual reasoning behind the two models is similar, the performance of both models on the data set is compared.

4.1. Rigid Canopies

[19] The best fit value of the (dimensionless) $K_r$ is 0.21 ± 0.04 for the rigid canopies. This estimate is robust with respect to the number of data points used to estimate its value. When half of the data points were randomly selected and used to estimate $K_r$, its value did not change markedly. This provides confidence that the estimate of $K_r$ is robust and can be applied to other conditions.

[20] Figures 3a and 3b compare measured and predicted values of $n$ and $U_b$ of the rigid canopy data set for the proposed model and for the model of Huthoff et al. [2007]. Both models perform reasonably well in estimating both variables. Within the data set of Stone [1997], four groups of experiments exist for which all runs have the same canopy density, canopy height, and flow depth and width. Only the channel slope (and flow rate) differs between runs. Because $n$ is independent of $S$, the estimated $n$ is the same for each of the runs in a group. However, the measured $n$ varies between estimates. These groups are identifiable as the clusters of points that form a horizontal line in Figure 3a. The estimates of $U_b$ for the same data points do not have a qualitatively different error structure than the rest of the data set, suggesting that the (in)sensitivity of the $n$ estimates to the channel slope $S$ is more incorrect than the sensitivity of the $U_b$ estimates to $S$. Since the only difference between the models of $n$ (equation (17)) and $U_b$ (equation (16)) is in the application of Manning’s equation, this in turn suggests that the variation in measured $n$ for these runs occurs because of an $S$ dependence in $n$, rather than because of some other variable factor between the different runs that is not accounted for here.

[21] Table 1 shows the coefficient of determination, root-mean-square error (RMSE), and bias for each model and variable combination. The model proposed here has a lower root-mean-square error than the model of Huthoff et al. [2007]. Although the eddy length scale and the ejection contribution used by the two models differ, errors associated with both models are broadly similar. Experiments with relatively high error in one model often have relatively high errors for the other. This can be explained by noting that the model performance is relatively insensitive to the length scale used for $r$ (i.e., the limiting vertical length scale). The root-mean square distance between estimates of $n$ with eddy length scale of $r$ and estimates using an eddy length scale equal to $S$, is only 0.0014 s m$^{-3}$.

[22] The model is relatively insensitive to the precise value of the drag coefficient used. When using the value $C_d = 1.13$ instead of the reported $C_d$ when available, no estimate of $n$ changed by more than 15% (not shown). In general, the drag coefficient may depend on the exact variation of canopy properties with height from the channel bottom, which may not be known and can be difficult to estimate for a large range of species without detailed measurements. The model’s insensitivity to $C_d$ thus significantly improves the range of its applicability for practical purposes.

4.2. Flexible Canopies

[23] For flexible canopies, the best fit value of $K_r$ is lower at 0.74 ± 0.016. The reduction in $K_r$ is expected because vegetation deflection in flexible canopies often leads to drag reductions. The model performance for cases of flexible canopies is also presented in Figure 3 and summarized in Table 1. Not surprisingly, the models perform worse than for the rigid case. The deflection of flexible
vegetation affects not only the canopy height and drag, but also the leaf area density and any coupling between the flow inside and above the vegetation. This is not explicitly accounted for in either of the two models.

As a result of the failure to account for canopy deflection, the assumption of constant leaf area density, drag coefficient, and canopy layer velocity $U_c$ is no longer realistic in flexible canopies. In reality, $U_c$ increases with height and changes during the deflection cycle. The tangent velocity scale $v_t$ is higher than $U_c$ at the bottom of the eddy, so that the value of $v_t$ used in the eddy momentum transfer, calculated as $(U_c + \Delta U) - U_c = \Delta U$, is too high. Equation (13) therefore predicts a $U_b$ that is too low, and the estimate of $n$ is high. This is reflected in Table 1 through the bias in $n$. The magnitude of this error is expected to depend on how far the dominant eddy can penetrate into the canopy, and therefore on the ratio of $r$ to $h_c$.

Table 1. Comparison Between Measured and Modeled $U_b$ and $n$ for Rigid and Flexible Canopies

<table>
<thead>
<tr>
<th></th>
<th>$R^2$</th>
<th>RMSE</th>
<th>Bias</th>
<th>$m$</th>
<th>$b$</th>
</tr>
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<tbody>
<tr>
<td>Rigid Canopy</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$n$, Huthoff et al.</td>
<td>0.80</td>
<td>0.019</td>
<td>0.81</td>
<td>0.55</td>
<td>0.013</td>
</tr>
<tr>
<td>$n$, proposed</td>
<td>0.82</td>
<td>0.012</td>
<td>1.09</td>
<td>0.57</td>
<td>0.026</td>
</tr>
<tr>
<td>$U_{\mu}$, Huthoff et al.</td>
<td>0.90</td>
<td>0.068</td>
<td>1.29</td>
<td>1.18</td>
<td>0.010</td>
</tr>
<tr>
<td>$U_{\mu}$, proposed</td>
<td>0.90</td>
<td>0.046</td>
<td>0.96</td>
<td>0.80</td>
<td>0.020</td>
</tr>
<tr>
<td>Flexible Canopy</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$n$, Huthoff et al.</td>
<td>0.65</td>
<td>0.021</td>
<td>1.51</td>
<td>0.69</td>
<td>0.027</td>
</tr>
<tr>
<td>$n$, proposed</td>
<td>0.65</td>
<td>0.019</td>
<td>1.33</td>
<td>0.50</td>
<td>0.028</td>
</tr>
<tr>
<td>$U_{\mu}$, Huthoff et al.</td>
<td>0.51</td>
<td>0.21</td>
<td>0.79</td>
<td>0.46</td>
<td>0.12</td>
</tr>
<tr>
<td>$U_{\mu}$, proposed</td>
<td>0.44</td>
<td>0.21</td>
<td>0.89</td>
<td>0.34</td>
<td>0.19</td>
</tr>
</tbody>
</table>

*The coefficient of determination ($R^2$), the root-mean-square error (RMSE), and the relative error or bias ($\text{bias} = (\text{measured} - \text{modeled})/\text{measured}$) are shown. Parameters $m$ and $b$ are the slope and intercept of a robust linear regression of the predicted against the measured values with a bilinear weighting scheme. Huthoff et al. refers to the work by Huthoff et al. [2007].

Figure 3. Comparison between predicted and measured (a) Manning’s roughness coefficient $n$ and (b) bulk velocity $U_b$ for the laboratory experiments with rigid canopy elements. (c and d) The same as Figures 3a and 3b, respectively, but showing the laboratory experiments with flexible canopy elements. Black symbols represent predictions of the proposed model, while green symbols represent those from the model of Huthoff et al. [2007]. Crosses represent points for which the drag coefficient was not studied in the original experiment and for which its value was assumed to equal 1.13 as in the work by Dunn et al. [1996].
with flexible canopies, except the data of Carolla et al. [2002]. Unlike other data sets, that of Carolla et al. [2002] uses a variety of species as canopy elements, likely complicating the model behavior and sensitivities. Note that, with the exception of data from Jarvela [2005] and Carolla et al. [2002], most estimates have a relative error of less than 0.5, a reasonable value given the large range of tested $n$.

As canopy density increases, the system begins to act similarly to flow over a permeable wall of height $h_c$. The flow rate below the depth to which the eddy penetrates becomes essentially zero. The contribution of $U_c$ to equation (9) should then be less, such that the model used here overestimates $U_b$ and underestimates $n$. This prediction is supported by the model underestimating $n$ for the nine data points of Jarvela [2005], which are far the least porous data points measured (taking canopy porosity as $1 - \frac{2}{3} \rho \bar{a} d$, [Nepf et al., 2004]). The velocity profiles observed in the study of Jarvela [2005] are near zero in much of the canopy, suggesting that the lack of flow is the dominant source of error for these nine experiments.

5. Conclusions

The model presented here phenomenologically links turbulence theory with vegetation properties to make predictions about flow in densely vegetated channels. The model can be applied to both rigid and flexible vegetation, but performs better for the rigid case as expected. The discrepancy in performance is attributed to the fact that the model does not fully account for the variability in the deflected vegetation height between sweeps and ejections [Ghisalberti and Nepf, 2006] or for the effect of the canopy deflection on the flow. When the vegetation canopy is flexible, $C_d$ and $U_c$ can vary significantly with height and $U_b$, which may contribute to prediction errors for flexible canopies. Vegetation and vegetation proxies are not perfectly cylindrical and may include leaves and other elements that cause significant variations in $C_d$ and $a$ with height. This amplifies the effects of any variability with height in $U_b$.

To use a phenomenological approach for describing the canopy shear stress (equation (1)), two quantities must be determined: the eddy velocity scaling $v_e$ and $v_t$ and the proportionality between the eddy velocities and the shear stress, given by $K_r$. The value of $K_r$ estimated here is smaller than values of $K_r$ proposed for similar phenomenological models [Gioia and Chakraborty, 2006; Gioia et al., 2010], both in the case of rigid and of flexible canopies. Simulation studies can provide further insight into the value and dependence of $K_r$ on vegetation properties. Significant progress has been made in recent years on simulating the dynamic two-way interaction between deflecting plants and flow using large-eddy simulation studies [Dupont et al., 2010], and other approaches [de Langre, 2008]. These approaches may also be used to determine the variables that might impact $K_r$, including the density and the elastic properties of the vegetation.

The magnitude of the bias and the RMSE for the model proposed here are lower than those generated by the model of Huthoff et al. [2007] for both rigid and flexible canopies. One of the key differences between the model of Huthoff et al. [2007] and that presented here is the choice of length scale as the scaling factor for the eddy velocity. When applied to rigid canopies, the model is essentially insensitive to the choice of a vertical ($r$) or horizontal ($S$) length scale. Though the choice of the eddy length scale influences the estimates of $n$ and $U_b$ for flexible canopies, the effect is still much smaller than the effect of the choice of $v_e$. Physically, the proposed model suggests ejections are nonnegligible contributors to the shear stress at the top of the canopy. The small sensitivity to $n$ and $U_b$ is not surprising, because equations (16) and (17) show that $U_b$ and $n$ depend only on $s^{1/3}$. This one-third power results from the use of Kolmogorov’s four-fifths law for scaling the eddy velocity and has the effect of dampening variations in the length scale. Despite this dampening effect, a vertical length scale performs slightly better than using a horizontal length scale.

Finally, the fact that the model presented here allows prediction of $n$ and $U_b$ independently of the spacing between canopy elements is a considerable advantage. It allows the model to be applied to heterogeneous canopies that do not have a unique spacing width, a necessary step toward bridging the gap between measurements in laboratory flows and predictions for natural channels.

Appendix A

The assumption that the turbulent spectrum is entirely in the inertial range can be relaxed to derive a more complete equation for $u_c$. This equation is derived

![Figure 4](image-url). The magnitude of relative error in the bulk velocity $U_b$ (the magnitude of the error in predicted $U_b$, normalized by the observed value) as a function of $0.21l_c/h_c$, the ratio of the predicted $r/h_c$ for different data sets. Only data with flexible canopies for which $r/h_c = 0.21l_c/h_c$ are shown. Thus, very sparse data sets (for which $(C_d h_c)^{-1} > 4$) and data sets for which the flow depth above the canopy is limiting ($h_w - h_c < \delta$) are not shown.
here in analogy to work of Gioia and Chakraborty [2006].
The u* depends on the energy spectrum according to
\[ u_*^2 = \frac{1}{\varepsilon} \frac{E(\sigma)}{\sigma^2} d\sigma, \]  
(A1)
where \( \sigma \) is a length scale, and \( E(\sigma) \) is given by [Pope, 2000]
\[ E(\sigma) = A e^{2/3} \sigma^{1/2} c_d(\sigma)c_c(\sigma), \]  
(A2)
where \( c_d \) and \( c_c \) are correction formulas for the reductions in the dissipative and energetic regions of the spectrum, respectively. \( A \) is a dimensionless constant and has a value of about 1.5 [Tennekes and Lumley, 1972, p. 271].

The \( c_d(\sigma) = \exp(-\beta_0^2/\sigma) \), where \( \beta = 2.1 \) scales the exponential and \( \eta = \nu^{3/4} e^{-1/4} \) is the viscous length scale, with \( \nu \) the kinematic viscosity of water. The value of \( \eta \) is determined below. The \( c_d \) decreases exponentially from one as \( \sigma \) decreases and nears the viscous length scale.
The \( c_c(\sigma) = (1 + \gamma (\sigma/R)^3)^{-1/6} \), where \( \gamma = 3.715 \) is a scaling constant [Pope, 2000, p. 232–234]. The \( c_c \) is \( \approx 1 \) for most \( \sigma \) but increases as \( \sigma \) nears \( R \).

[31] Kolmogorov’s four-fifths law is applied at the channel-wide scale to determine \( \epsilon \) as before, \( v_*^2 = (4/5) \epsilon R \). Considering the second-order structure function of velocity and assuming that the correlation function of the velocity fluctuations \( \omega \) goes to zero for large eddies, \( v_*^2 \sim R^2 \). Since the velocity fluctuations and the depth-averaged velocity scale with the same canopy boundary conditions, it is reasonable to assume \( u^2 \sim U_b^2 \). Defining the constant of proportionality between the two quantities to be \( K_u \), \( v_*^2 = K_u U_b \). Thus, \( \epsilon = 5/4K_u U_b^3/\epsilon \), which allows calculation of \( \eta \) as
\[ \eta = \nu^{3/4} \left( \frac{S K_u U_b^3}{4R^2} \right)^{1/2}. \]  
(A3)

[32] Combining these definitions and substituting into equation (A1) leads to
\[ u_*^2 = \left[ \frac{5}{4} \right] \frac{A}{R^{2/3}} \frac{1}{\varepsilon} \frac{1}{\sigma^{1/3}} \exp \left( -\frac{\beta_0^2}{\sigma} \right) \left( 1 + \gamma (\sigma/R)^2 \right)^{-1/6} d\sigma \]  
\times K_u^2 U_b^2. \]  
(A4)

Note that unlike in equation (6), \( u_* \) no longer depends linearly on \( U_b \) because \( \epsilon \) depends (slightly) on \( U_b \). This equation for \( u_* \) has an additional fitting parameter \( K_u \). If the equation is combined with the two-layer flow model described above, and an optimal fitting value of \( K_u \) is used, no estimate of \( n \) changes by more than 5%.

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References

Fenzl, R. (1962), Hydraulic resistance of broad shallow vegetated channels, PhD thesis, Univ. of Calif, Davis.


G. G. Katul, A. G. Konings, and S. E. Thompson, Nicholas School of the Environment, Duke University, Durham, NC 27708, USA (konings@alum.mit.edu)