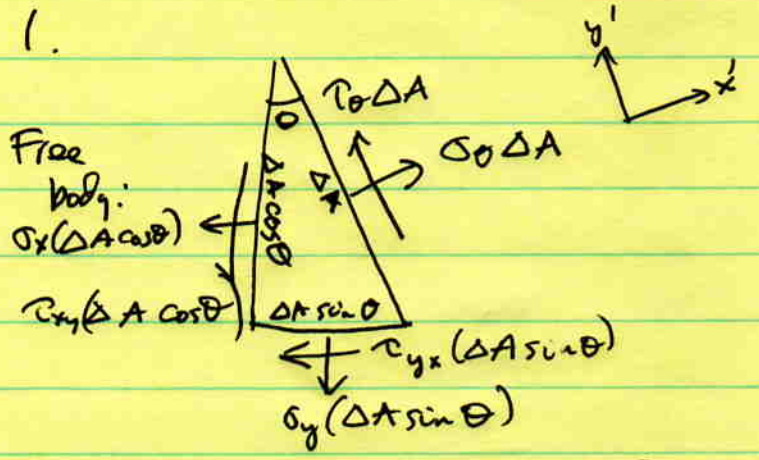
Solutions to Stress Problems

1.



$$\Sigma F_{x'} = 0 = \sigma_\theta \Delta A - \sigma_x(\Delta A\cos\theta)\cos\theta - \tau_{xy}(\Delta A\cos\theta)\sin\theta - \sigma_y(\Delta A\sin\theta)\sin\theta - \tau_{yx}(\Delta A\sin\theta)\cos\theta$$

$$\rightarrow \sigma_\theta = \sigma_x\cos^2\theta + \sigma_y\sin^2\theta + \underbrace{(\tau_{xy}+\tau_{yx})}_{=2\tau_{xy} \text{ because } \tau_{xy}=\tau_{yx}}\sin\theta\cos\theta$$

$$\Sigma F_{y'} = 0 = \tau_\theta \Delta A + \sigma_x(\Delta A\cos\theta)\sin\theta - \tau_{xy}(\Delta A\cos\theta)\cos\theta + \tau_{yx}(\Delta A\sin\theta)\sin\theta - \sigma_y(\Delta A\sin\theta)\cos\theta$$

$$\rightarrow \tau_\theta = \tau_{xy}(\cos^2\theta - \sin^2\theta) + (\sigma_y - \sigma_x)\sin\theta\cos\theta$$

2. $\sigma_\theta = \left(10\cos^2 30^\circ - 6\sin^2 30^\circ + 2\times 4\times \sin 30^\circ\cos 30^\circ\right)\text{ksi} = 9.46 \text{ ksi}$

$\tau_\theta = \left[4(\cos^2 30^\circ - \sin^2 30^\circ) + (-6-10)\sin 30^\circ\cos 30^\circ\right)\text{ksi} = -4.93 \text{ ksi}$

3. $\tau_{xy}=0 \rightarrow \tau_\theta = (\sigma_y - \sigma_x)\sin\theta\cos\theta = \frac{\sigma_y-\sigma_x}{2}\sin 2\theta$

$|\tau_\theta| = \frac{|\sigma_y-\sigma_x|}{2}\sin 2\theta$, maximum at $\theta = \pm 45^\circ$

$$\tau_{max} = \frac{|\sigma_y-\sigma_x|}{2}$$