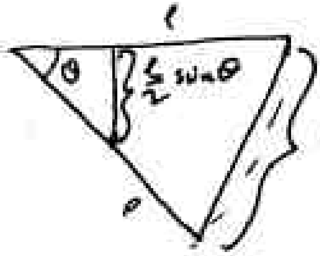
E36, Section 2, Spring 2005
Homework 15 Solutions
7/38,40,42,46,49

7/38



$2l\sin\frac{\theta}{2}$ = stretch of spring

$$V_g = -mg\frac{l}{2}\sin\theta \quad , \quad V_e = \frac{1}{2}k\left(2l\sin\frac{\theta}{2}\right)^2$$

$$V = 2kl^2\sin^2\frac{\theta}{2} - mg\frac{l}{2}\sin\theta$$

$m = 60$ kg
$k = 160$ N/m
$l = 1.4$ m

but $2\sin^2\frac{\theta}{2} = 1-\cos\theta$

$$\therefore V = kl^2(1-\cos\theta) - mg\frac{l}{2}\sin\theta$$

$$\frac{dV}{d\theta} = kl^2\sin\theta - mg\frac{l}{2}\cos\theta$$

$$\theta = \tan^{-1}\frac{mg}{2kl} = \tan^{-1}\frac{60\times 9.81}{2\times 160\times 1.4} = 52.7^\circ$$

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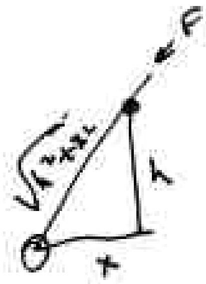

$$\delta s = \delta\sqrt{h^2+x^2} = \frac{x\,\delta x}{\sqrt{h^2+x^2}}$$

$$\delta U' = F\delta s \quad ; \quad V = \frac{1}{2}kx^2$$

$$\delta U = \delta U' - \delta V$$
$$= \left(\frac{Fx}{\sqrt{h^2+x^2}} - kx\right)\delta x = 0$$

$\rightarrow F = k\sqrt{h^2+x^2}$

$x = \sqrt{(F/k)^2 - h^2}$ provided $F > kh$

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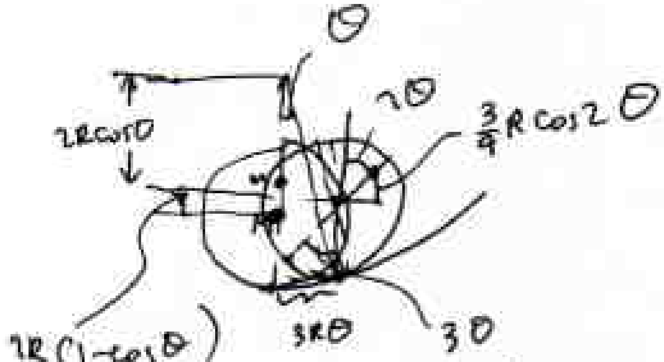

$$V = Mg\cdot 2R(1-\cos\theta) + mg\left[2R(1-\cos\theta) + \frac{3}{4}R\cos 2\theta\right]$$

$$\frac{d^2V}{d\theta^2} = (M+m)g\cdot 2R\cos\theta - 3mgR\cos 2\theta$$

$$\left.\frac{d^2V}{d\theta^2}\right|_{\theta=0} = (2M-m)gR > 0 \text{ for } \underline{m < 2M}$$

7/46 stretch $= 2b\cos\frac{\theta}{2} \rightarrow V_e = \frac{1}{2}k(2b)\cos^2\frac{\theta}{2}$

$V_m = mgb\sin\frac{\theta}{2}$, $V_P = P\cdot 2b\sin\frac{\theta}{2}$

$$V = 2kb^2\cos^2\frac{\theta}{2} + (mg+2P)b\sin\frac{\theta}{2}$$

$$2\frac{dV}{d\theta} = -4kb^2\cos\frac{\theta}{2}\sin\frac{\theta}{2} + (mg+2P)b\cos\frac{\theta}{2} = 0$$

$$\sin\frac{\theta}{2} = \frac{mg+2P}{4kb}, \quad \underline{\theta = 2\sin^{-1}\frac{mg+P}{4kb}}$$

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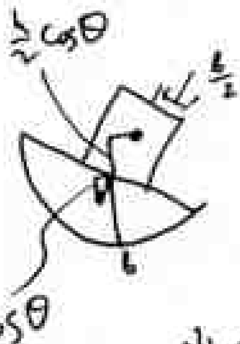

$$V = \rho g\left(\frac{\pi b^2}{4}h\cdot\frac{h}{2}\cos\theta - \frac{2}{3}\pi b^3\cdot\frac{3b}{8}\cos\theta\right)$$

$$= \frac{\rho g\pi b^2}{8}(h^2 - 2b^2)\cos\theta$$

$$\left.\frac{d^2V}{d\theta^2}\right|_{\theta=0} = \frac{\rho g\pi b^2}{8}(2b^2 - h^2) > 0 \text{ for } \underline{h < \sqrt{2}b}$$