University of California at Berkeley
Department of Civil and Environmental Engineering
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Engineering 36
Section 2
Spring 2005

Midterm Examination II

Solutions

Name ______________________
Last, First

Problem 1	___ /40
Problem 2	___ /60
Total	___ /100

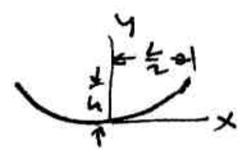
1. A symmetrically suspended weightless cable carries a load that varies like a half sine wave across the span, that is, if the span is L and x is measured from midspan, then

$$w(x) = w_0 \cos \frac{\pi x}{L}.$$

If the sag is h, find the maximum tension in the cable in terms of $w_0 L$ and h/L.

Formulas : $\quad T_0 \dfrac{d^2 y}{dx^2} = w(x), \qquad \dfrac{d}{d\theta} \cos\theta = -\sin\theta, \qquad \dfrac{d}{d\theta} \sin\theta = \cos\theta.$

$$\frac{d^2y}{dx^2} = \frac{w_0}{T_0} \cos \frac{\pi x}{L} \rightarrow \frac{dy}{dx} = \frac{w_0 L}{\pi T_0} \sin \frac{\pi x}{L}$$



$$\rightarrow y = \frac{w_0 L^2}{\pi^2 T_0} \left(1 - \cos \frac{\pi x}{L}\right)$$

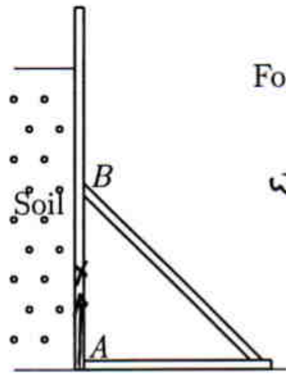
$$h = y\left(\frac{L}{2}\right) = \frac{w_0 L^2}{\pi^2 T_0} \rightarrow T_0 = \frac{w_0 L^2}{\pi^2 h}$$

$$|y'|_{max} = \frac{w_0 L}{\pi T_0} = \frac{\pi h}{L}$$

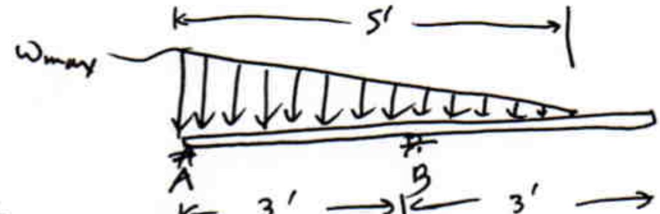
$$T_{max} = T_0 \sqrt{1 + y'^2_{max}}$$

$$= \frac{w_0 L}{\pi h / L} \sqrt{1 + \left(\frac{\pi h}{L}\right)^2}$$

2. The retaining wall whose cross-section is shown in the figure is made out of 6-inch-wide planks, 6 feet long, that are braced in the back at the bottom and at mid-height. If the soil is up to 5 feet, find the shear and bending-moment diagrams for each plank treated as a beam, with the brace connections A and B as simple supports, and with the soil as a fluid weighing 100 lb/ft^3. Neglect the thickness of the bracing.



Formulas : $p = \rho g h, \quad \frac{dV}{dx} = -w, \quad \frac{dM}{dx} = V$



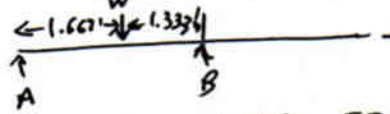
$\rho g = 100 \text{ lb/ft}^3$

$w_{max} = p_{max} \cdot 6 \text{ in} = 100 \frac{\text{lb}}{\text{ft}^3} \times 5 \text{ ft} \times 0.5 \text{ ft} = 250 \text{ lb/ft}$

$x < 5'$: $\quad w = w_{max}\left(1 - \frac{x}{5 \text{ ft}}\right) = 250 \frac{\text{lb}}{\text{ft}} - 50 \text{ lb} \cdot \frac{x}{\text{ft}^2}$

Equivalent concentrated load: $W = \frac{1}{2} w_{max} \times 5 \text{ ft} = 625 \text{ lb}$

Reactions:

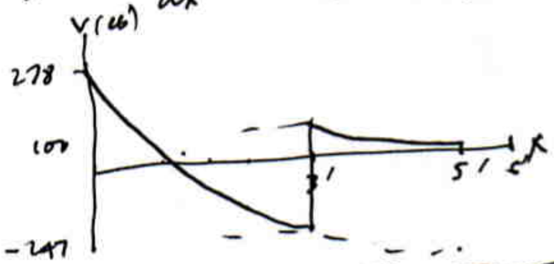


$\rightarrow A = \frac{1.333}{3} W = 278 \text{ lb}, \quad B = \frac{1.667}{3} W = 347 \text{ lb}$

Shear: $\frac{dV}{dx} = -250 \frac{\text{lb}}{\text{ft}} - 50 \frac{\text{lb}}{\text{ft}^2} x \Rightarrow V = \underbrace{V_0}_{= 278 \text{ lb}} - 250 \text{ lb} \frac{x}{\text{ft}} + 25 \frac{\text{lb}}{\text{ft}^2} x^2 \quad , \quad x < 3'$

$= 625 \text{ lb} - 250 \text{ lb} \frac{x}{\text{ft}} + 25 \frac{\text{lb}}{\text{ft}^2} x^3, \quad 3' < x < 5'$

$= 0, \quad 5' < x < 6'$



To find where $V = 0$:

$25x^2 - 250x + 278 = 0$

$x^2 - 10x + 11.11 = 0 \rightarrow x = 5 - \sqrt{5^2 - 11.11} = 1.273'$

Moment: $\frac{dM}{dx} = 278 \text{ lb} - 250 \text{ lb} \frac{x}{\text{ft}} + 25 \frac{\text{lb}\, x^2}{\text{ft}^2} \quad , \quad x < 3'$

$\Rightarrow M = \left(278 \frac{x}{\text{ft}} - 125 \frac{x^2}{\text{ft}^2} + \frac{25}{3} \frac{x^3}{\text{ft}^3}\right) \text{ lb-ft} \quad$ "

$M_{max}: M(1.273') = 168.2 \text{ lb-ft}; \quad M(3') = -66.7 \text{ lb-ft}$