

Added Exercises

Consider a scalar field $f(\mathbf{x})$, a vector field $\mathbf{v}(\mathbf{x})$, and a second order tensor field $\mathbf{T}(\mathbf{x})$. For each of the expressions below, write its indicial form.

1. $\nabla(\mathbf{T} \cdot \mathbf{v}) \rightarrow (T_{ij}v_j)_{,k} = T_{ij,k}v_j + T_{ij}v_{j,k}$
2. $\nabla^2 \mathbf{T} \rightarrow T_{ij,kk}$
3. $\nabla \cdot (f\mathbf{v}) \rightarrow (fv_i)_{,i} = f_{,i}v_i + fv_{i,i}$
4. $\nabla \times \nabla \times \mathbf{T} \rightarrow e_{pqi}e_{klj}T_{ij,lq}$
5. $\nabla^2(\nabla \times \mathbf{v}) \rightarrow e_{ijk}v_{k,jmm}$
6. $\nabla^4 f \rightarrow f_{,iijj}$
7. $\nabla(\mathbf{x}\mathbf{v}) \rightarrow (x_iv_j)_{,k} = \delta_{ik}v_j + x_iv_{j,k}$
8. $\nabla \cdot (\mathbf{x} \cdot \mathbf{T}) \rightarrow (x_iT_{ij})_{,j} = \delta_{ij}T_{ij} + x_iT_{ij,j} = T_{ii} + x_iT_{ij,j}$
9. $\nabla \cdot \mathbf{x} \rightarrow x_{i,i} = \delta_{ii} = 3$
10. $\nabla \cdot (\mathbf{v}\mathbf{x}) \rightarrow (v_ix_j)_{,j} = v_{i,j}x_j + 3v_i$
11. $\nabla \cdot [(\nabla \cdot \mathbf{v})\mathbf{x}\mathbf{x}] \rightarrow [v_{i,i}x_jx_k]_{,k} = v_{i,ik}x_jx_k + v_{i,i}x_j + 3v_{i,i}x_j = v_{i,ik}x_jx_k + 4v_{i,i}x_j$
12. $\nabla \times (f\mathbf{v}) \rightarrow e_{ijk}(fv_k)_{,j} = e_{ijk}(f_{,j}v_k + fv_{k,j})$
13. $\nabla(\mathbf{T} \cdot \mathbf{T}^T) \rightarrow (T_{ij}T_{kj})_{,l} = T_{ij,l}T_{kj} + T_{ij}T_{kj,l}$