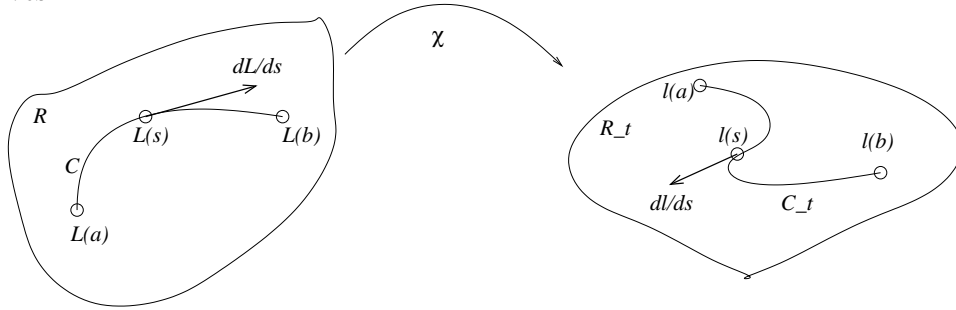


Mappings of tangents to curves

In lecture we showed that the deformation gradient, F_{iA} , maps 'local' vectors from the reference configuration to the current (or spatial) configuration. The result is approximate in the sense that $F_{iA}dX_A = dx_i + \text{h.o.t.}$ ¹. In all of our constructions we always focus on the properties at a point and thus implicitly always consider the limit as $dX_A \rightarrow 0$. Sometimes, however, it is useful to have an expression for the mapping of certain vectors independent of their magnitudes. Such vectors are the tangent vectors to material curves.



Consider a reference configuration curve \mathcal{C} defined by the points $\mathbf{L} : [a, b] \rightarrow \mathcal{R}$, where $[a, b]$ is an arbitrary interval of the real line, \mathbb{R} . At each point $s \in [a, b]$ of the curve, the tangent vector to the curve is given by $d\mathbf{L}(s)/ds$. After deformation, every point of \mathcal{C} is mapped to \mathcal{C}_t by the deformation map, thus defining a new curve $\mathbf{l} : [a, b] \rightarrow \mathcal{R}_t$, where $\mathbf{l}(s) = \chi(\mathbf{L}(s))$. The tangent vector at each point of this new curve is given by:

$$\begin{aligned} \frac{d\mathbf{l}}{ds} &= \frac{\partial \chi}{\partial \mathbf{X}} \cdot \frac{d\mathbf{L}}{ds} & \frac{dl_i}{ds} &= \frac{\partial \chi_i}{\partial X_A} \frac{dL_A}{ds} \quad (1) \\ &= \mathbf{F} \cdot \frac{d\mathbf{L}}{ds} & &= F_{iA} \frac{dL_A}{ds} \quad (2) \end{aligned}$$

From this last result, we can observe that *without approximation of any form!* the tangent vector to a curve of material points in the reference configuration is mapped by the deformation gradient to a tangent vector of a curve in the spatial configuration composed of the same material points. For

¹h.o.t. means higher order terms and in this context terms that are quadratic or higher in the norm of dX_A .

this reason, in some books you will find the deformatino gradient referred to as the tangent map.