

HW #2 Solutions: CE231 / MSE211

1.

$$(\mathbf{a} \cdot \mathbf{b})\mathbf{c} = (2*0 - 5*8 - 7*1)\mathbf{c} = -47*(4\mathbf{e}_1 + 5\mathbf{e}_2 + 7\mathbf{e}_3) = -188\mathbf{e}_1 - 235\mathbf{e}_2 - 329\mathbf{e}_3 \rightarrow \begin{pmatrix} -188 \\ -235 \\ -329 \end{pmatrix} \quad (1)$$

2.

$$b_3\mathbf{a} \cdot \mathbf{c} = 1 * (4 * 2 + 5 * 5 - 7 * 7) = -16 \quad (2)$$

3.

$$(\mathbf{a} \cdot \mathbf{b})\mathbf{a} \otimes \mathbf{b} = -47(a_i b_j \mathbf{e}_i \otimes \mathbf{e}_j) = \quad (3)$$

$$\begin{aligned} -47 & \left(0\mathbf{e}_1 \otimes \mathbf{e}_1 - 16\mathbf{e}_1 \otimes \mathbf{e}_2 + 2\mathbf{e}_1 \otimes \mathbf{e}_3 \right. \\ & + 0\mathbf{e}_2 \otimes \mathbf{e}_1 - 40\mathbf{e}_2 \otimes \mathbf{e}_2 + 5\mathbf{e}_2 \otimes \mathbf{e}_3 \\ & \left. + 0\mathbf{e}_3 \otimes \mathbf{e}_1 + 56\mathbf{e}_3 \otimes \mathbf{e}_2 - 7\mathbf{e}_3 \otimes \mathbf{e}_3 \right) \end{aligned} \quad (4)$$

$$\rightarrow -47 \begin{bmatrix} 0 & -16 & 2 \\ 0 & -40 & 5 \\ 0 & 56 & -7 \end{bmatrix} \quad (5)$$

4. $\delta_{ii} = 1 + 1 + 1 = 3$

5. $T_{3j}\delta_{3j} = T_{33} = 3$

6. $T_{ij}\delta_{ij} = T_{11} + T_{22} + T_{33} = 7$

7.

$$\begin{aligned} T_{ij}T_{ij} &= T_{11}^2 + T_{12}^2 + T_{13}^2 \\ &+ T_{21}^2 + T_{22}^2 + T_{23}^2 \\ &+ T_{31}^2 + T_{32}^2 + T_{33}^2 \end{aligned} \quad (6)$$

$$= 1 + 64 + 4 + 64 + 9 + 4 + 4 + 4 + 9 = 163 \quad (7)$$

8.

$$\mathbf{T}\mathbf{c} = T_{ij}(\mathbf{e}_i \otimes \mathbf{e}_j)c_k \mathbf{e}_k = T_{ik}c_k \mathbf{e}_i = 58\mathbf{e}_1 + 61\mathbf{e}_2 + 39\mathbf{e}_3 \rightarrow \begin{pmatrix} 58 \\ 61 \\ 39 \end{pmatrix} \quad (8)$$

9. $I_T = T_{ii} = 7$

10. $II_T = \frac{1}{2}(T_{ii}^2 - T_{ij}T_{ji}) = \frac{1}{2}(7 * 7 - 163) = -57$

11. $III_T = 1 * (3 * 3 - 2 * 2) - 8 * (8 * 3 - 2 * 2) + 2 * (8 * 2 - 3 * 2) = -135$

12.

$$\mathbf{e}_1 \otimes \mathbf{e}_2 \rightarrow \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (9)$$

13.

$$\mathbf{e}_3 \otimes \mathbf{e}_2 \rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad (10)$$