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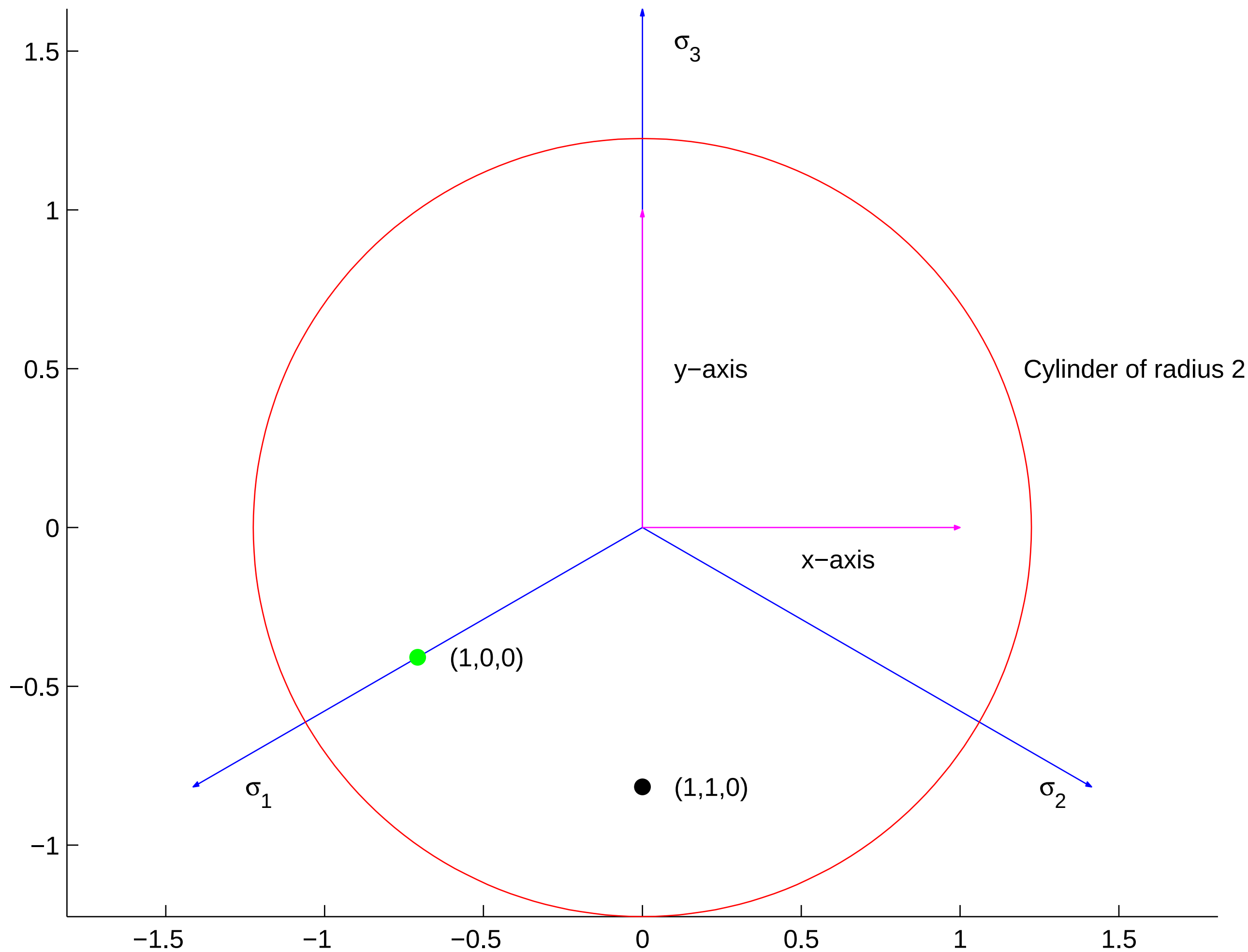
Geometry of the π-plane

In the isotropic setting, a state of stress can be visualized in terms of its eigenvalues; i.e. it can plotted as a point in $\mathbb{R}^3$ where the coordinate axes represent the three principal values. The deviatoric part of the stress is then given by the three (principal) values $(s_1, s_2, s_3) = (\sigma_1, \sigma_2, \sigma_3) - \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)(1, 1, 1)$. Deviatoric stresses in this setting satisfy the constraint $s_1 + s_2 + s_3 = 0$ and thus are representable by points in a two dimensional space. This two dimensional space is the π-plane.

For effective visualization of deviatoric quantities one needs to understand the basic geometry of the π-plane. The plane is one with a principal stress space normal in the $(1, 1, 1)$ direction. When the coordinate axes (the axes of principal stress values) are projected into this plane they appear as 3 rays separated by $2\pi/3$ rad (or 120 degrees). A projection of an arbitrary stress state into the π-plane is given by $\vec{s} = \vec{\sigma} - (\vec{\sigma} \cdot \vec{n})\vec{n}$ where $\vec{n} = (1, 1, 1)/\sqrt{3}$. If we take the third principal axis as the 'vertical' axis in the plane then the unit vector in the vertical direction is given by $\vec{y} = (-1/3, -1/3, 2/3)/\sqrt{6/9}$. The 'horizontal' direction in the plane is then given by $\vec{x} = \vec{y} \times \vec{n} = (-1, 1, 0)/\sqrt{2}$. Thus given an arbitrary stress state $\vec{\sigma}$, its coordinates in the π-plane are given by $(\vec{x} \cdot \vec{\sigma}, \vec{y} \cdot \vec{\sigma})$.

In the figure below we show the projections of the three principal axes, the horizontal and vertical axes (of the π-plane), as well as the intersection of the π-plane with a von Mises cylinder of radius 2, and two different states of stress projected onto the plane.



1.5
1
0.5
0
−0.5
−1
−1.5
−1
−0.5
0
0.5
1
1.5
σ3
σ1
σ2
y−axis
x−axis
Cylinder of radius 2
(1,0,0)
(1,1,0)