On Determining Wood Thermal Diffusivity and Probe Geometry Using In-Situ Heat Response Curves from Sap Flow Measurements

Xingyuan Chen¹, Gretchen Miller¹, Dennis Baldocchi², Yoram Rubin¹

¹Department of Civil and Environmental Engineering, University of California, Berkeley, CA 94720
²Department of Environmental Science, Policy, and Management, University of California, Berkeley, CA 94720
chenxy@berkeley.edu

Abstract
The heat pulse method is widely used to measure water flux through plants; it works by inferring the velocity of water through a porous medium from the speed at which a heat pulse is propagated through the system. No systematic, non-destructive calibration procedure exists to determine the site-specific parameters necessary for calculating sap velocity, e.g., wood thermal diffusivity and probe geometry. Such parameter calibration is crucial to obtain the correct transpiration flux density from the sap flow measurements at the plant scale; and consequently, to the up-scale of tree-level water flux to the canopy and landscape scales.

The purpose of this study is to present a statistical framework to estimate wood thermal diffusivity and probe geometry simultaneously using in-situ measurements of temperature traces following a heat pulse. The parameters are inferred using a Bayesian inversion technique, based on the Markov chain Monte Carlo sampling method. The primary advantage of the proposed methodology is that it does not require known probe geometry or any further intrusive sampling of sapwood. The Bayesian framework also enables direct quantification of uncertainty in estimated sap flow velocity.

Experiments using synthetic data show that multiple tests on the same apparatus are essential to obtain reliable and accurate solutions. When applied to field conditions, these tests are conducted during different seasons and seasonality of wood thermal diffusivity is obtained as a by-product of the parameter estimation process. Empirical factors are introduced to account for the influence of non-ideal probe geometry on the estimation of heat pulse velocity, and they are estimated as well. The proposed methodology is ready to be applied to calibrate existing heat ratio sap flow systems at other sites. It is especially useful when alternative transpiration calibration devices such as lysimeter are not available.

I. Introduction

Site Information

- Located on the lower foothill of the Sierra Nevada
- Annual precipitation ~ 560mm
- Mean annual air temperature ~ 16.6°C
- Hot and dry in summer, cold and wet in winter
- Deciduous oak trees coexist with annual grasses
- Eddy covariance system installed
- Sap flow measurements on representative trees

Fundamentals of Sap Flow Measurements

- Heat pulse velocity: \( v_h = \frac{\Delta T}{\Delta T_x} \)
- Sap velocity: \( v_s = \frac{\rho v}{a\rho c_v} \)
- Sap flux density: \( J_s = a v_s \)
- Sap flux: \( Q_s = J_s A \)

II. Theoretical Background

Heat Transport Equation

\[
\frac{\partial T}{\partial t} - \kappa \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) = q + \frac{Q(x,y,z,t)\delta(x)\delta(y)\delta(z)}{\rho c_v}.
\]

Idealized Solution (Marshall, 1958)

- Assume infinite medium and line heater, instantaneous heating

\[
\Delta T(x,y,t) = \frac{q}{4\pi\rho c_v} \exp \left( \frac{-(x-v_h t)^2 + y^2}{4\kappa t} \right)
\]

Improved Solution

- Assume finite dimension of medium and heater, pulsed heating

\[
\Delta T(x,y,z,t) = \frac{q}{4\pi\rho c_v} \exp \left( \frac{-(x-v_h t)^2 + y^2}{4\kappa t} \right) \int_{-L}^{L} \left[ \sin \left( b_s z \right) \cos \left( b_s z \right) \right] ds
\]

The difference between the idealized solution and improved solution is only significant for times shortly after the heat pulse

Heat Pulse Velocity

- Symmetric probe alignment: \( v_h = \frac{\kappa \ln \frac{\Delta T}{\Delta T_x}}{x} \)
- Asymmetric probe alignment: \( v_h = B_s + B_x \frac{\kappa \ln \frac{\Delta T}{\Delta T_x}}{x} \)

with \( B_s \) and \( B_x \) dependent on probe geometry