

Name:

M.S Comprehensive Examination

Analysis

Note:

1. Dimensions, properties and loading are given in consistent units in all problems.
2. All figures are drawn to scale.
3. Calculations should be shown in detail with all intermediate steps; it is recommended to manipulate expressions symbolically as far as possible and substitute numbers only at or near the end.

Formulas

The following relation holds between the flexural basic forces \mathbf{q} and the flexural deformations \mathbf{v} of a homogeneous, prismatic beam element of length L and flexural stiffness EI :

$$\mathbf{v} = \mathbf{f} \mathbf{q} \quad \text{with} \quad \mathbf{f} = \frac{L}{6EI} \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

The inverse of the flexural flexibility matrix \mathbf{f} gives the flexural stiffness matrix \mathbf{k} of a homogeneous, prismatic beam element of length L and flexural stiffness EI :

$$\mathbf{k} = \frac{2EI}{L} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

The deformations \mathbf{v}_0 of a homogeneous, prismatic beam element of length L and flexural stiffness EI under a uniformly distributed load w are

$$\mathbf{v}_0 = \frac{wL^3}{24EI} \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

The symbolic inverse of a 2x2 matrix \mathbf{M} is

$$\mathbf{M} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \rightarrow \mathbf{M}^{-1} = \frac{1}{\det(\mathbf{M})} \begin{bmatrix} M_{22} & -M_{12} \\ -M_{21} & M_{11} \end{bmatrix} \quad \text{with} \quad \det(\mathbf{M}) = M_{11}M_{22} - M_{12}M_{21}$$

1. Problem (50% weight)

The homogeneous, prismatic simply supported girder in Fig. 1 with flexural stiffness EI is subjected to a uniform load w over the left half of its span.

You are asked to answer the following questions:

1. Determine the maximum bending moment value in terms of w and L and draw the bending moment diagram.
2. Determine the vertical translation at midspan in terms of w , L and EI .

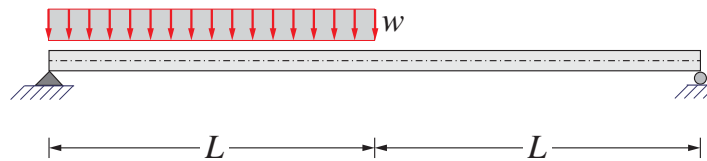


Figure 1: Simply supported girder

2. Problem (50% weight)

Fig. 2 shows the structural model for a column-girder assembly consisting of the column element a and the girder element b. Both frame elements can be assumed *inextensible* with flexural stiffness EI . The structural model is subjected to a concentrated horizontal force P_h at node 2.

You are asked to answer the following questions:

1. Determine the horizontal translation at the point of load application in terms of P_h , L and EI .
2. Determine the bending moment at the ends of the column in terms of P_h and L and draw the bending moment diagram for the column-girder assembly.

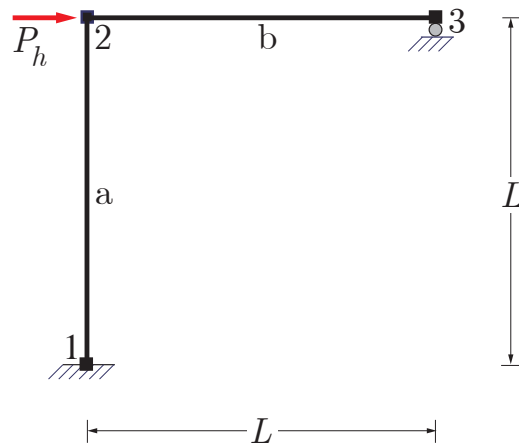


Figure 2: Structural model for column-girder assembly

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4. Results involving multiplication or division with a matrix larger than 2 x 2 will not receive credit.

Formulas

The following relation holds between the basic forces \mathbf{q} and the deformations \mathbf{v} of a homogeneous, prismatic beam element of length L and flexural stiffness EI :

$$\mathbf{v} = \mathbf{f}\mathbf{q} \quad \text{with} \quad \mathbf{f} = \frac{L}{6EI} \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

The deformations \mathbf{v}_0 of a homogeneous, prismatic beam element of length L and flexural stiffness EI under a uniformly distributed load w are

$$\mathbf{v}_0 = \frac{wL^3}{24EI} \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

1. Problem (50% weight)

The *inextensible* simply supported girder in Fig. 1 consists of two segments with a change of flexural stiffness at midspan: the left half of the girder has flexural stiffness $2EI$ and the right half has flexural stiffness EI . The right half of the girder is subjected to a uniformly distributed load w .

You are asked to answer the following questions:

1. Determine the bending moment distribution and draw the bending moment diagram as precisely as possible supplying in particular the maximum value.
2. Determine the vertical translation at midspan.

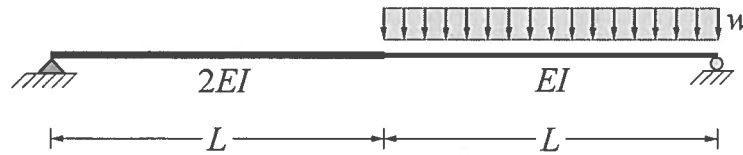


Figure 1: Simply supported girder with flexural stiffness change at midspan under uniform load w

Note: The problem is suitable for a symbolic solution, but if you prefer numerical results, use $L = 5$, $w = 6$, and $EI = 80,000$.

2. Problem (50% weight)

The *inextensible* propped cantilever in Fig. 2 consists of two segments with a change of flexural stiffness at midspan: the left half of the girder has *infinite flexural stiffness* while the right half has flexural stiffness EI . The right half of the girder is subjected to a uniformly distributed load w .

You are asked to answer the following questions:

1. Determine the bending moment distribution and draw it as precisely as possible supplying in particular the maximum value.
2. Determine the vertical translation at midspan.

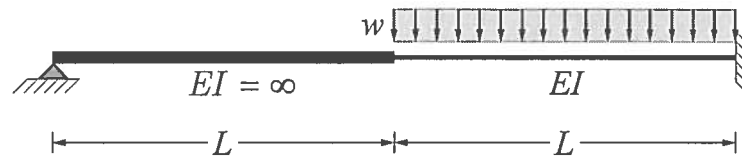


Figure 2: Propped cantilever with flexural stiffness change at midspan under uniform load w

Note: The problem is suitable for a symbolic solution, but if you prefer numerical results, use $L = 5$, $w = 6$, and $EI = 80,000$.

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