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**Ph.D. Preliminary Examination**  
**Analysis**

**Note:**

1. Dimensions, properties and loading are given in consistent units in all problems.
2. All figures are drawn to scale.
3. Calculations should be shown in detail with all intermediate steps; it is recommended to manipulate expressions symbolically as far as possible and substitute numbers only at or near the end.

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**1. Problem (50% weight)**

The continuous beam over two spans in Fig. 1 is subjected to a concentrated force  $P_v$  at the middle of each span. The left span of length  $2L$  has flexural stiffness  $EI_1$  and the second span also of length  $2L$  has flexural stiffness  $EI_2$ .

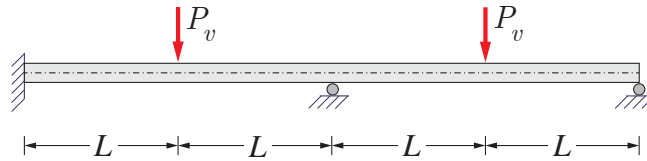


Figure 1: Continuous beam over two spans

The analysis of the continuous beam over two spans under the given load pattern gives a bending moment in the middle of the left span of  $0.23992P_vL$  and a bending moment in the middle of the right span of  $0.35484P_vL$ .

You are asked to answer the following questions:

1. Determine the stiffness ratio  $EI_1/EI_2$ .
2. Determine the vertical deflection under the concentrated force of the left span in terms of  $P_v$ ,  $L$  and  $EI_1$ .

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## 2. Problem (50% weight)

The structural model in Fig. 2 consists of five frame elements a through e. The frame elements have very large axial capacity and the following flexural capacity values  $M_p$ : elements a, b and e have flexural capacity  $M_p$  of 250 units, element c has flexural capacity  $M_p$  of 300 units, and element d has flexural capacity  $M_p$  of 350 units. Fig. 2 shows the location of the plastic hinges at incipient collapse: they form at both ends of element a, end  $j$  of element b, both ends of element c, and end  $j$  of element e noting that end  $i$  corresponds to the lower numbered node to which the element connects.

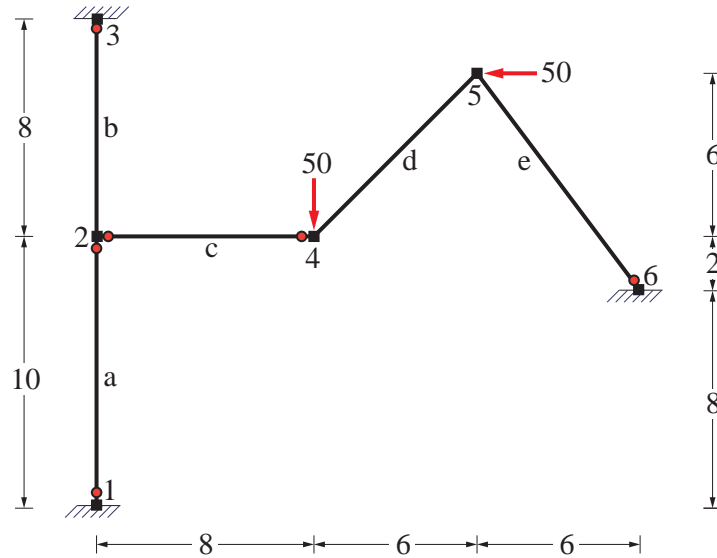


Figure 2: Simply supported girder with flexibly supported overhang

You are asked to answer the following questions:

1. Determine the collapse load factor  $\lambda_c$  under the given load pattern.
2. Draw the bending moment diagram of the structure at incipient collapse in Fig. 3 and supply all bending moment values in the figure.
3. Determine the vertical support reaction at node 6.

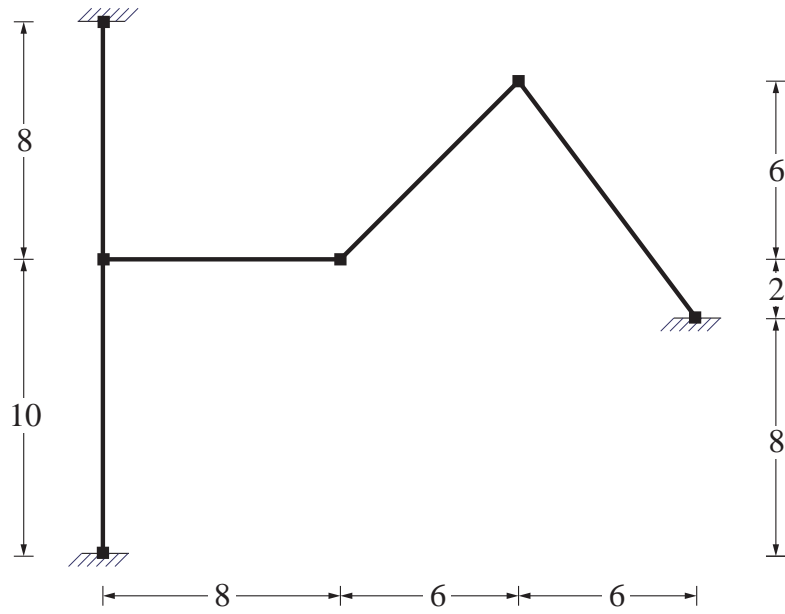


Figure 3: Bending moment diagram

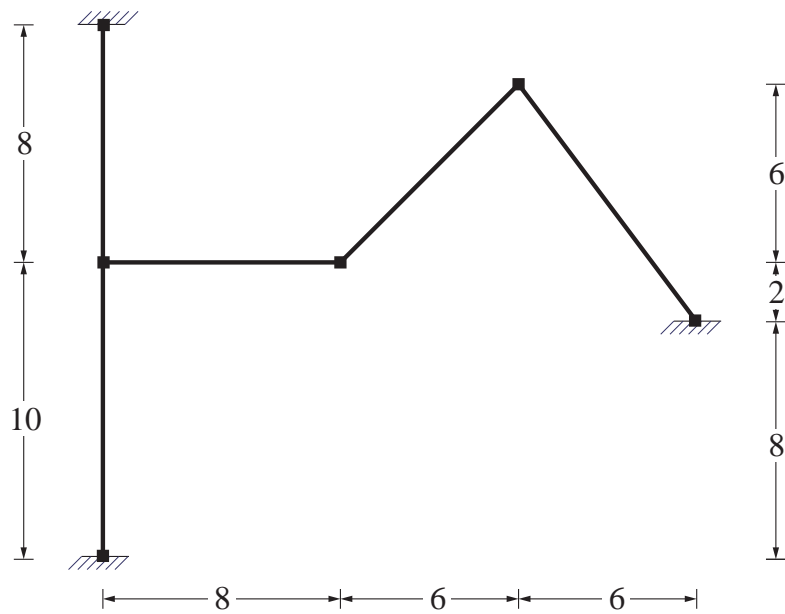


Figure 4: Auxiliary figure

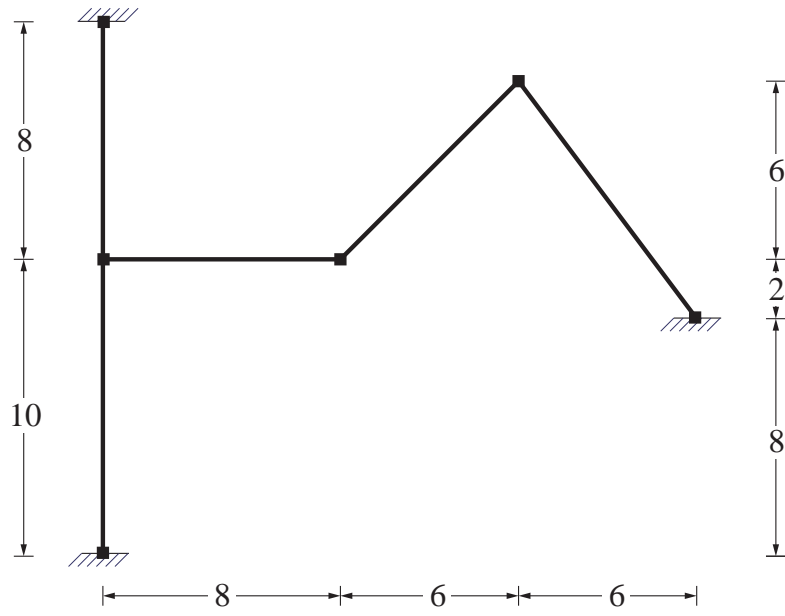


Figure 5: Auxiliary figure

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### 1. Problem (50% weight)

The structural model in Fig. 1 of a braced frame consists of three *inextensible* frame elements a, b, and c and a truss (brace) element d. The flexural section stiffness  $EI$  of the frame elements a, b and c is 100,000, while the axial section stiffness  $EA$  of the brace element d is 40,000 units.

The structure is subjected to a downward vertical nodal force of 50 units at node 2, as Fig. 1. In addition, the brace element d is prestressed with an initial prestressing force  $q_0 = 100$ .

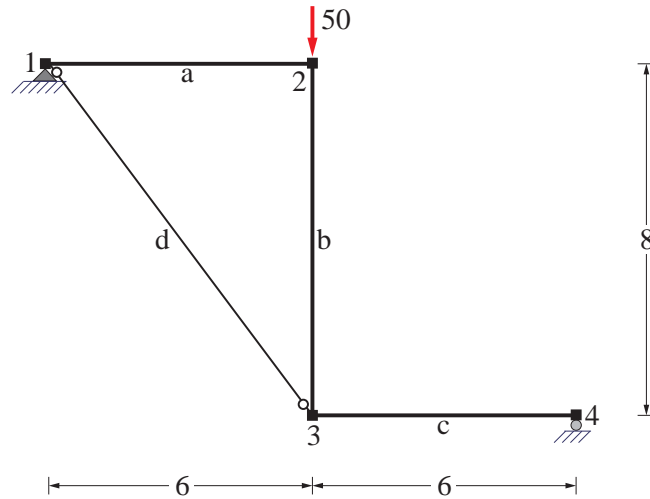


Figure 1: Braced frame geometry and loading

You are asked to answer the following questions:

1. Draw the bending moment distribution in Fig. 2.
2. Determine the node translations and draw the deformed shape in Fig. 3.

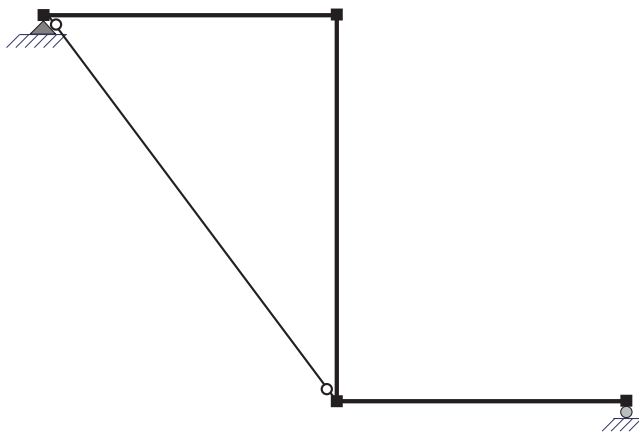


Figure 2: Bending moment distribution

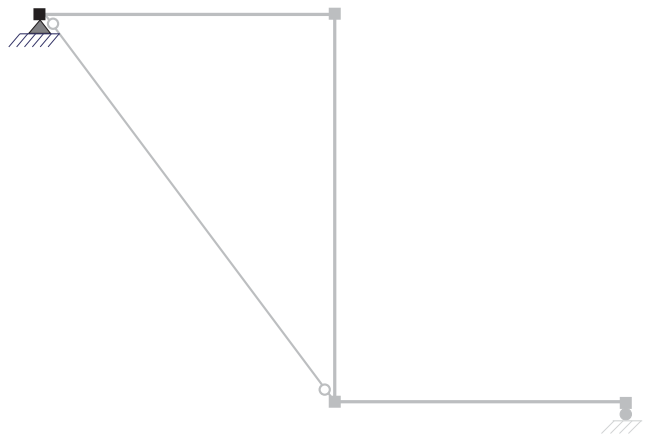


Figure 3: Deformed shape

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## 2. Problem (50% weight)

The continuous girder in Fig. 4 over two equal spans of length  $2L$  is subjected to a uniform element load  $w$  that extends over the left half of each span, as Fig. 4 shows. The continuous girder has uniform flexural stiffness  $EI$ .

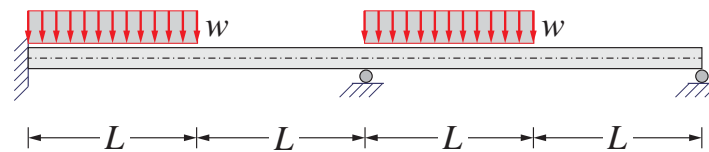


Figure 4: Two span continuous girder geometry and loading

You are asked to answer the following questions:

1. Propose a suitable discrete structural model for determining the bending moment distribution of the continuous girder under consideration of the rule that division and multiplication with matrices larger than  $2 \times 2$  is not allowed for hand calculations.
2. Determine and draw the bending moment distribution under the given loading in Fig. 5.
3. Determine the vertical translation at the middle of the right span of the continuous girder.

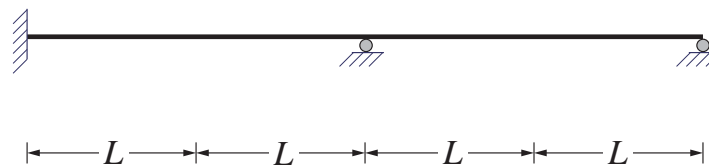


Figure 5: Bending moment distribution



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**1. Problem (50% weight)**

The structural model in Fig. 1 consists of three *inextensible* frame elements a, b, and c and a truss element d. The frame elements a and c are also *inflexible*. The structure is subjected to a uniformly distributed element load  $w = 10$  in element b. The flexural stiffness  $EI$  of element b is 80,000 units, while the axial stiffness  $EA$  of element d is 10,000 units. The truss element d is *prestressed* with an initial force  $q_0 = 20$ .

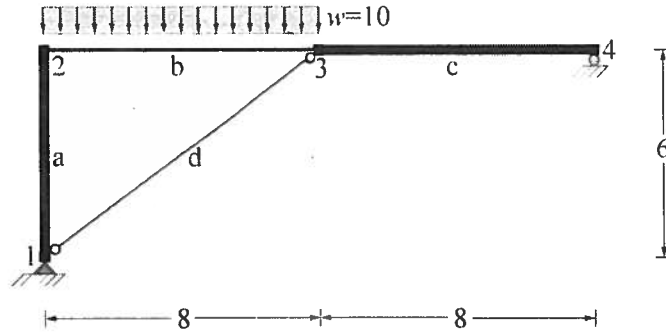


Figure 1

You are asked to answer the following questions:

1. Determine the horizontal translation at node 2.
2. Determine the bending moment distribution and draw the bending moment diagram as precisely as possible supplying in particular the maximum value.

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## 2. Problem (50% weight)

The simply supported girder with overhang in Fig. 2 has a flexible support of axial stiffness  $k_s = 2,000$  at the tip of the overhang. It carries a concentrated force of 50 units at the midspan of the simply supported portion. The girder is inextensible with uniform flexural stiffness  $EI$  of unknown value. The length  $L$  in Fig. 2 is also not available.

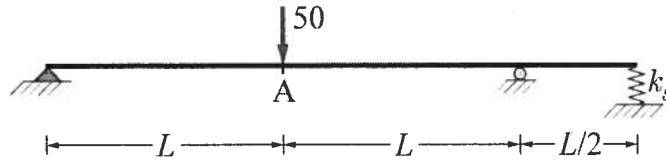


Figure 2: Simply supported girder with flexibly supported overhang

You are asked to answer the following question:

1. Determine the bending moment distribution and draw the bending moment diagram in terms of  $L$  knowing that the concentrated force of 10 units at the tip of the overhang in Fig. 3 causes an *upward translation* of the midspan point A by  $1.25 \cdot 10^{-2}$  units and a *downward translation* of the point B of load application by  $2.083 \cdot 10^{-2}$  units.

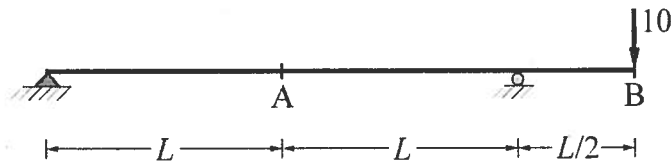


Figure 3: Simply supported girder with overhang under force at the tip

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1. Problem (50% weight)

The continuous beam over two spans of equal length  $L$  in Fig. 1 has a flexible middle support with axial stiffness  $k_s$ . It is subjected to a vertical force  $P_v$  at the middle of the second span. The beam has uniform flexural stiffness  $EI$ .

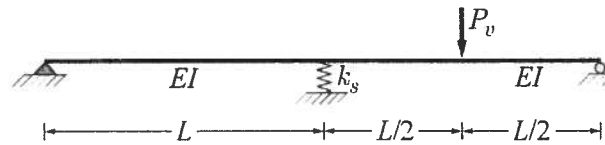


Figure 1: Continuous beam with flexible middle support

You are asked to answer the following questions:

1. What is the value of the spring stiffness  $k_s$  in terms of the span length  $L$  and the flexural stiffness  $EI$ , if the axial force in the spring is 65% of the applied force  $P_v$ ?
2. For the spring stiffness value of the preceding question determine the vertical translation at the point of load application.

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**2. Problem (50% weight)**

The structural model in Fig. 2 consists of 2 *inextensible* frame elements a and d with flexural stiffness  $EI$  and of 2 *inextensible and inflexible* elements b and c. The structure is subjected to a uniformly distributed load  $w$  of 10 units in elements a and b, as Fig. 2 shows.

You are asked to answer the following questions:

1. Determine the vertical translation at node 2 in terms of  $EI$ .
2. Draw the bending moment diagram under the given loading.

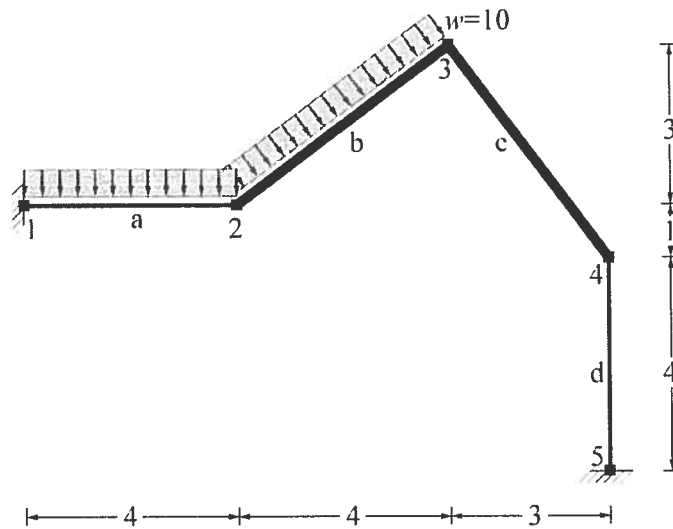


Figure 2

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1. Problem (50% weight)

The structural model in Fig. 1 consists of three *inextensible* frame elements a, b, and c.

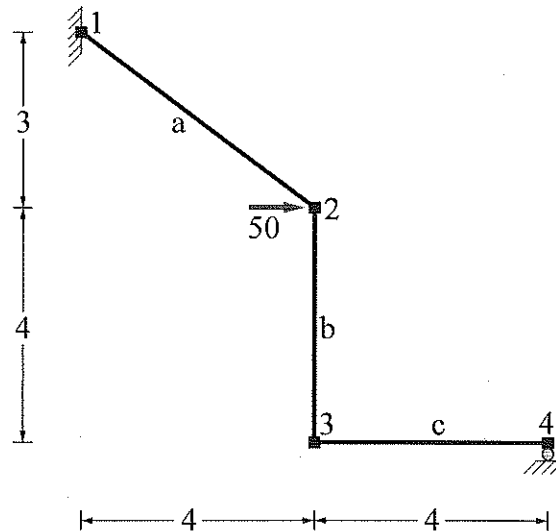


Figure 1: Frame with linear elastic, perfectly plastic basic force-deformation relation

The frame elements exhibit a linear elastic, perfectly plastic basic force-deformation relation with very high plastic axial capacity  $N_p$ . The flexural stiffness  $EI$  of the frame elements is equal to 40,000 units. The frame element a has plastic flexural capacity  $M_p$  of 80 units, while the plastic flexural capacity  $M_p$  of the elements b and c is 100 units. *It is assumed that the presence of an axial force in the frame elements does not affect their plastic flexural capacity.*

The structure is subjected to a horizontal nodal force of 50 units at node 2, as Fig. 1 shows.

You are asked to answer the following questions:

1. Determine the load factor  $\lambda_1$  at the formation of the first plastic hinge.
2. Determine the collapse load factor  $\lambda_c$ .
3. Draw the bending moment distribution  $M(x)$  at incipient collapse.

**Note:** Questions (2) and (3) can be solved independently of the first.



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## 2. Problem (50% weight)

The structural model in Fig. 2 consists of 3 frame elements a, b and c, and two truss elements d and e. The frame elements are *inextensible*. In addition, the elements a and c are *inflexible*. The frame element b has flexural stiffness  $EI$  of 10,000 units. The truss element d has axial stiffness  $EA$  of 10,000 units, and the truss element e has axial stiffness  $EA$  of 5,000 units. The truss element d is prestressed with an initial prestressing force  $q_0$  of -10 units (compressive force).

The structural model is subjected to a uniformly distributed load  $w$  of 10 units in element c, as Fig. 2 shows.

You are asked to answer the following questions:

1. Determine the vertical translation at node 2.
2. Determine the basic element forces in all elements.
3. Draw the bending moment diagram under the given loading.

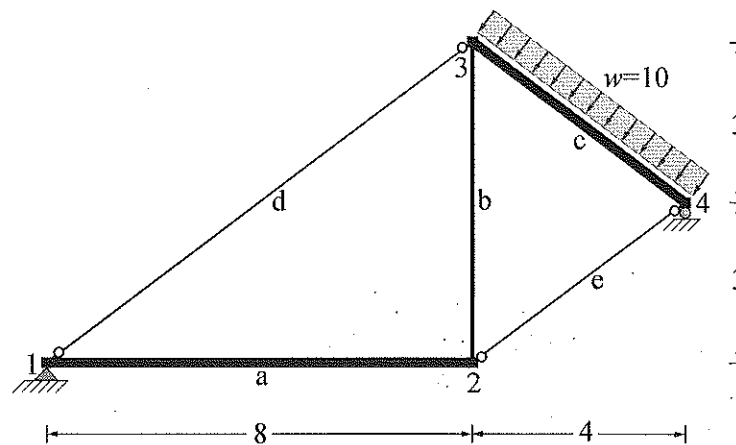


Figure 2