

Name:

Ph.D. Preliminary Examination
Analysis

Note:

1. Dimensions, properties and loading are given in consistent units in all problems.
2. All figures are drawn to scale.
3. Calculations should be shown in detail with all intermediate steps; it is recommended to manipulate expressions symbolically as far as possible and substitute numbers only at or near the end.

1. Problem (50% weight)

The tied frame model in Fig. 1 consists of 3 *inextensible* frame elements a, b and c with flexural stiffness EI and a tie (truss) element d with axial stiffness EA .

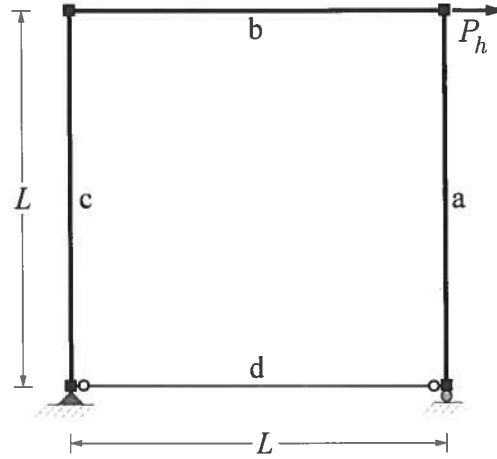


Figure 1: Tied two hinge frame

You are asked to answer the following questions in terms of L and the stiffness ratio EA/EI :

1. What is the influence coefficient relating the axial force in the tie d to the applied horizontal force P_h ?
2. What is the influence coefficient relating the horizontal translation at the roller to the applied horizontal force P_h ? What is the common name for such an influence coefficient?

2. Problem (50% weight)

The two hinge parabolic arch in Fig. 2 carries a vertical force P_v and a horizontal force P_h at its apex. It has span l and height h with $h = l/4$. You are asked to determine the collapse load of the arch under the assumption that it possesses uniform plastic capacity M_p that is independent of the axial force. It is proposed to use the structural model in Fig. 3 to approximate the geometry of the parabolic arch: it consists of four *inextensible*, straight frame elements with flexural capacity M_p connecting nodes on the parabola of the ideal arch axis.

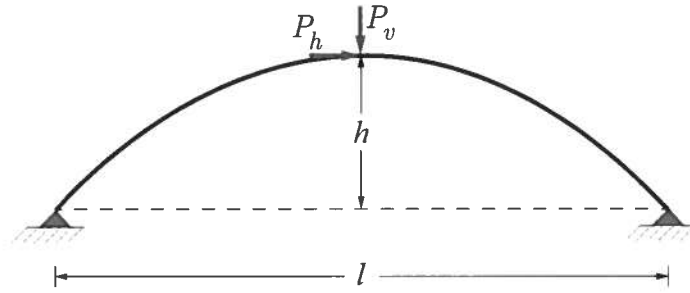


Figure 2: Two hinge parabolic arch

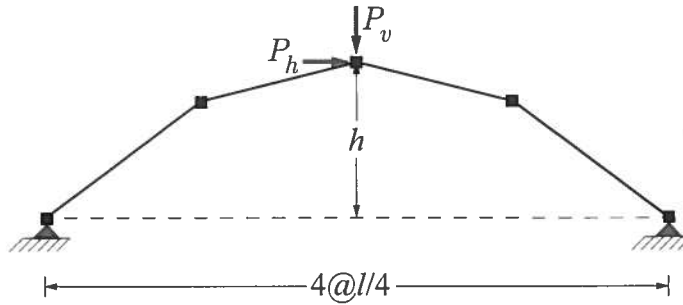


Figure 3: Idealization of parabolic arch with 4 straight frame elements

You are asked to answer the following questions:

1. Determine the load factor λ_c of the structural model in Fig. 3 in terms of the span l , the ratio $\eta = P_h/P_v$ in the range $0 \leq \eta \leq 1$ and the plastic flexural capacity M_p . You can assume that the nodal forces P_h and P_v act in the direction shown in the figures.
2. Do you expect the result of the preceding question to be larger or smaller than the exact load factor for the parabolic arch in Fig. 2? Explain why.
3. Describe two methods for improving the estimate of the collapse load factor for the parabolic arch in Fig. 2 with a discrete structural model. While calculations are not required, you are asked to provide sufficient detail, so that any required calculations could be set up with relative ease.

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1. Problem (50% weight)

The one-story braced frame in Fig. 1 consists of 2 frame elements a and b and a truss (brace) element c. The frame elements are assumed to be inextensible and have flexural section stiffness EI of 50,000. The axial stiffness of EA of the brace element is 50,000. The frame is subjected to a horizontal force at node 2 of 50 units acting to the left. In addition, the brace element c is prestressed, but the initial prestressing force q_0 was not recorded.

Determine the prestressing force q_0 so that the brace element c is not deformed under the given loading.

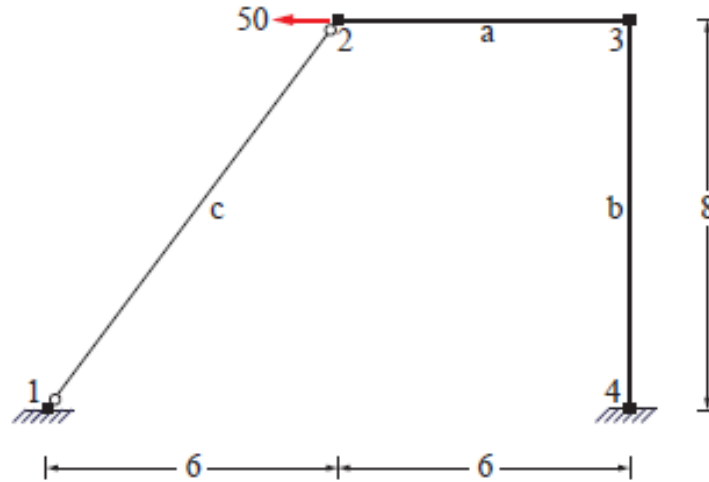


Figure 1: Structural geometry and loading

2. Problem (50% weight)

The structure in Fig. 2 carries a *reference* downward vertical force of 50 units and a *reference* horizontal force of 80 units at node 3. The plastic flexural capacity M_p of elements a and c is 180 units and the plastic flexural capacity M_p of elements b, d, e is 200 units. The plastic axial capacity N_p of the brace element f is 30 units. Elements a through e are assumed inextensible and have very large plastic axial capacity. The flexural stiffness EI of the frame elements a through e is 200,000 units and the axial stiffness EA of the brace element f is 10,000 units. The plastic hinge distribution at incipient collapse is given in Fig. 2.

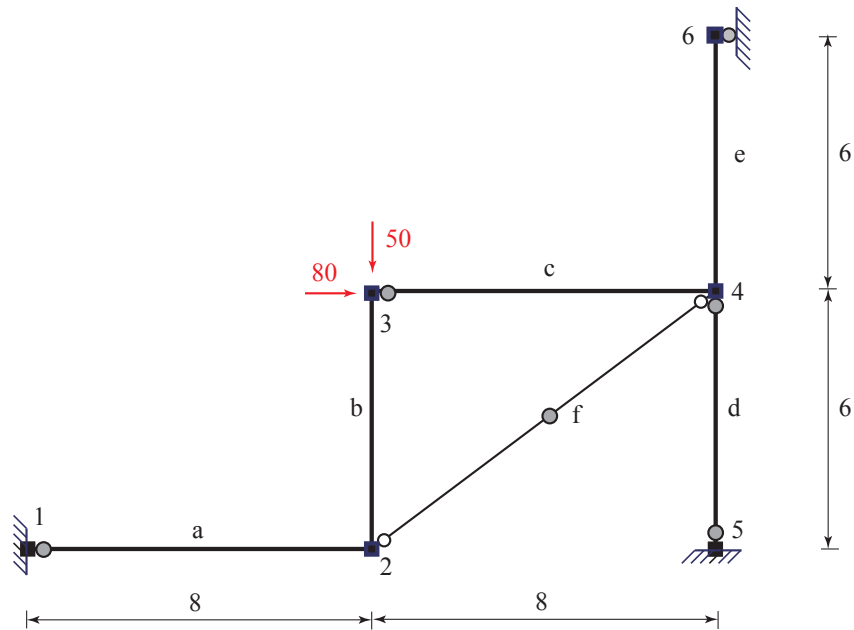


Figure 2: Structure geometry, and loading, and plastic hinge locations

You are asked the following question:

1. Determine the collapse load factor λ_c of the structure under the given loading.
2. Draw the bending moment distribution indicating clearly which way the frame elements bend.

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1. Problem (50% weight)

The continuous beam over two spans in Fig. 1 is subjected to a concentrated force P_v at the middle of each span. The left span of length $2L$ has flexural stiffness EI_1 and the second span also of length $2L$ has flexural stiffness EI_2 .

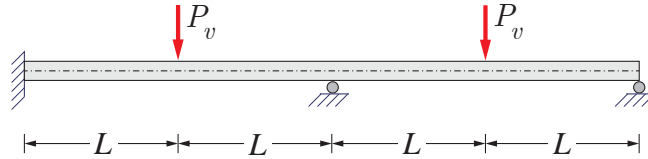


Figure 1: Continuous beam over two spans

The analysis of the continuous beam over two spans under the given load pattern gives a bending moment in the middle of the left span of $0.23992P_vL$ and a bending moment in the middle of the right span of $0.35484P_vL$.

You are asked to answer the following questions:

1. Determine the stiffness ratio EI_1/EI_2 .
2. Determine the vertical deflection under the concentrated force of the left span in terms of P_v , L and EI_1 .

2. Problem (50% weight)

The structural model in Fig. 2 consists of five frame elements a through e. The frame elements have very large axial capacity and the following flexural capacity values M_p : elements a, b and e have flexural capacity M_p of 250 units, element c has flexural capacity M_p of 300 units, and element d has flexural capacity M_p of 350 units. Fig. 2 shows the location of the plastic hinges at incipient collapse: they form at both ends of element a, end j of element b, both ends of element c, and end j of element e noting that end i corresponds to the lower numbered node to which the element connects.

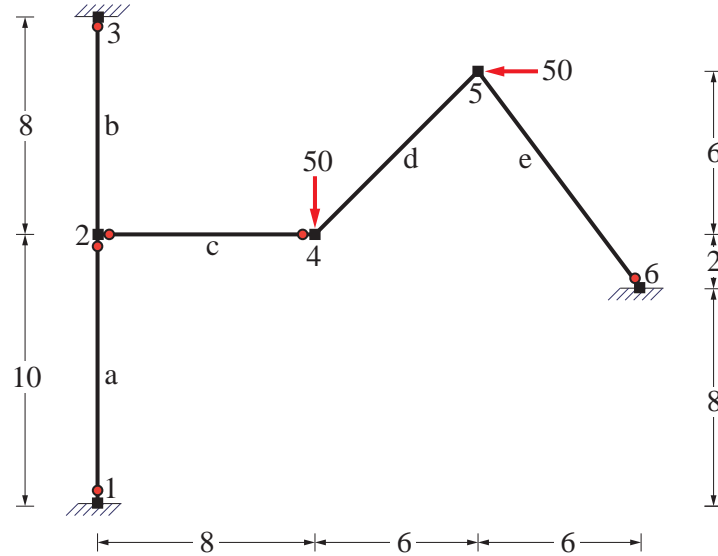


Figure 2: Simply supported girder with flexibly supported overhang

You are asked to answer the following questions:

1. Determine the collapse load factor λ_c under the given load pattern.
2. Draw the bending moment diagram of the structure at incipient collapse in Fig. 3 and supply all bending moment values in the figure.
3. Determine the vertical support reaction at node 6.

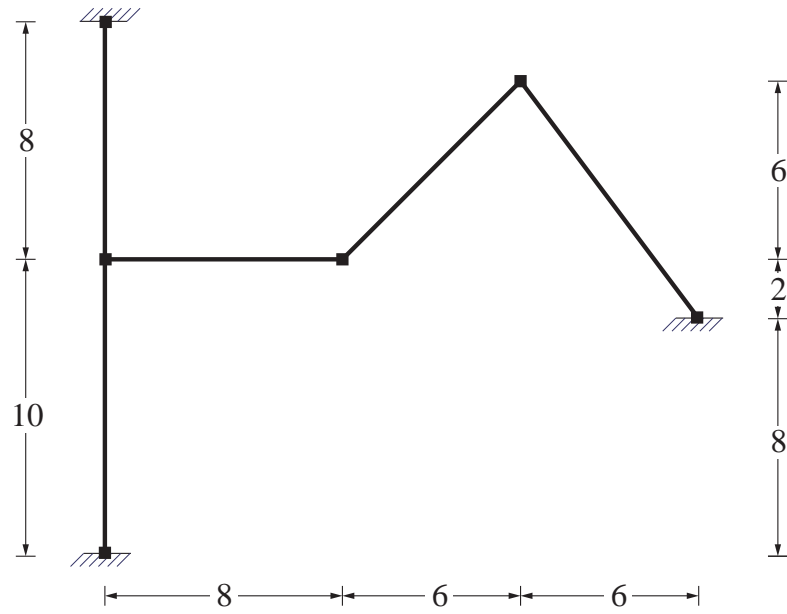


Figure 3: Bending moment diagram

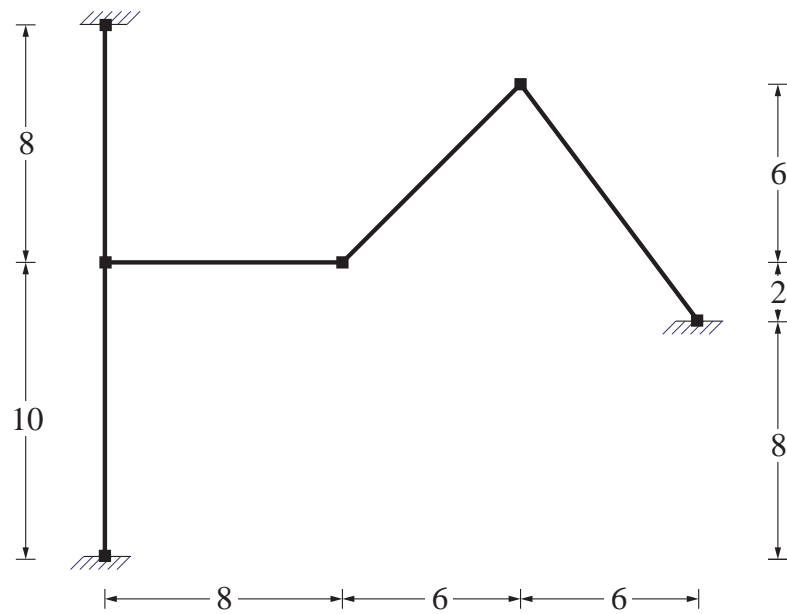


Figure 4: Auxiliary figure

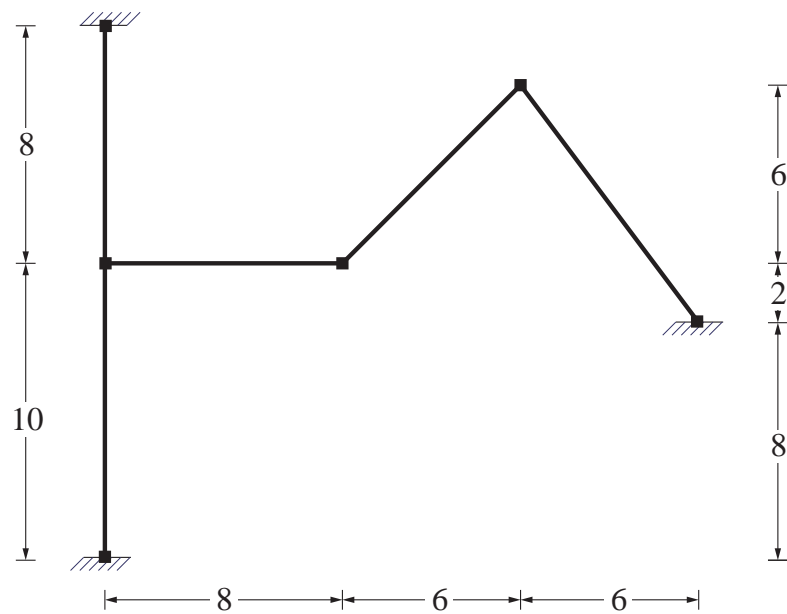


Figure 5: Auxiliary figure

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4. Results involving multiplication or division with a matrix larger than 2×2 will not receive credit.

1. Problem (50% weight)

The structural model in Fig. 1 of a braced frame consists of three *inextensible* frame elements a, b, and c and a truss (brace) element d. The flexural section stiffness EI of the frame elements a, b and c is 100,000, while the axial section stiffness EA of the brace element d is 40,000 units.

The structure is subjected to a downward vertical nodal force of 50 units at node 2, as Fig. 1. In addition, the brace element d is prestressed with an initial prestressing force $q_0 = 100$.

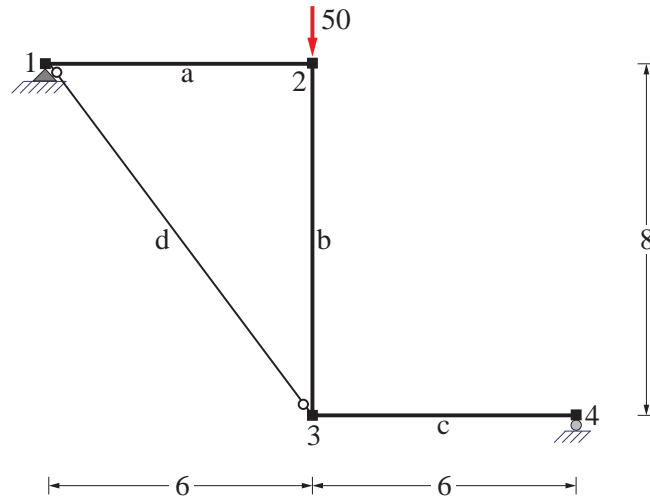


Figure 1: Braced frame geometry and loading

You are asked to answer the following questions:

1. Draw the bending moment distribution in Fig. 2.
2. Determine the node translations and draw the deformed shape in Fig. 3.

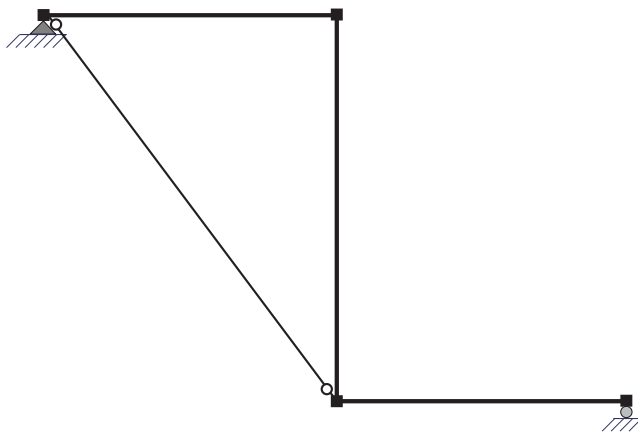


Figure 2: Bending moment distribution

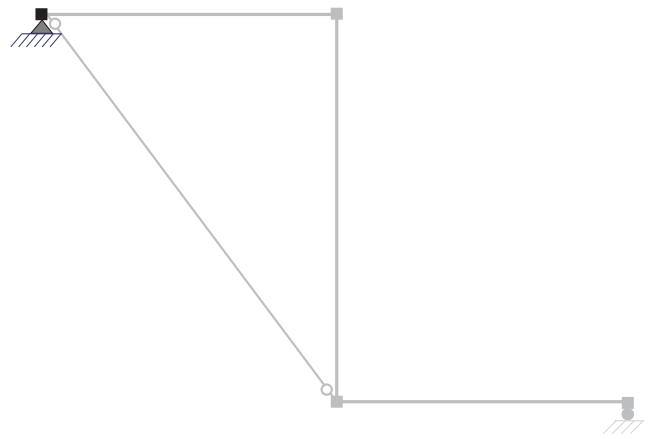


Figure 3: Deformed shape

2. Problem (50% weight)

The continuous girder in Fig. 4 over two equal spans of length $2L$ is subjected to a uniform element load w that extends over the left half of each span, as Fig. 4 shows. The continuous girder has uniform flexural stiffness EI .

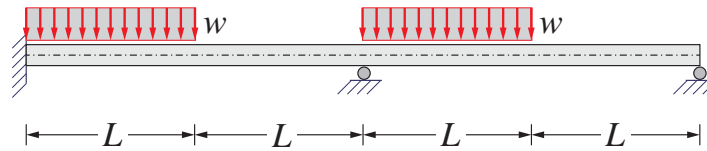


Figure 4: Two span continuous girder geometry and loading

You are asked to answer the following questions:

1. Propose a suitable discrete structural model for determining the bending moment distribution of the continuous girder under consideration of the rule that division and multiplication with matrices larger than 2×2 is not allowed for hand calculations.
2. Determine and draw the bending moment distribution under the given loading in Fig. 5.
3. Determine the vertical translation at the middle of the right span of the continuous girder.

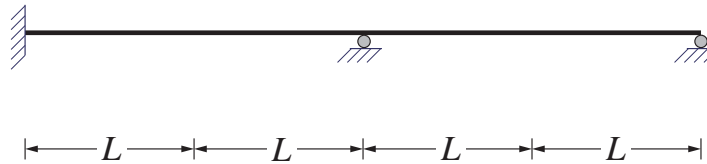


Figure 5: Bending moment distribution

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4. Results involving multiplication or division with a matrix larger than 2×2 will not receive credit.

1. Problem (50% weight)

The structural model in Fig. 1 consists of three *inextensible* frame elements a, b, and c and a truss element d. The frame elements a and c are also *inflexible*. The structure is subjected to a uniformly distributed element load $w = 10$ in element b. The flexural stiffness EI of element b is 80,000 units, while the axial stiffness EA of element d is 10,000 units. The truss element d is *prestressed* with an initial force $q_0 = 20$.

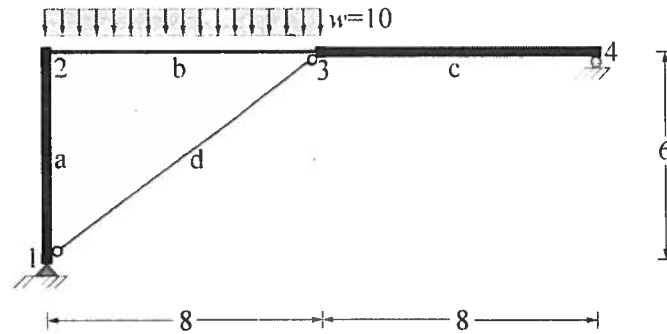


Figure 1

You are asked to answer the following questions:

1. Determine the horizontal translation at node 2.
2. Determine the bending moment distribution and draw the bending moment diagram as precisely as possible supplying in particular the maximum value.

2. Problem (50% weight)

The simply supported girder with overhang in Fig. 2 has a flexible support of axial stiffness $k_s = 2,000$ at the tip of the overhang. It carries a concentrated force of 50 units at the midspan of the simply supported portion. The girder is inextensible with uniform flexural stiffness EI of unknown value. The length L in Fig. 2 is also not available.

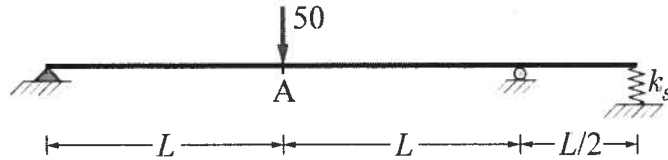


Figure 2: Simply supported girder with flexibly supported overhang

You are asked to answer the following question:

1. Determine the bending moment distribution and draw the bending moment diagram in terms of L knowing that the concentrated force of 10 units at the tip of the overhang in Fig. 3 causes an *upward translation* of the midspan point A by $1.25 \cdot 10^{-2}$ units and a *downward translation* of the point B of load application by $2.083 \cdot 10^{-2}$ units.

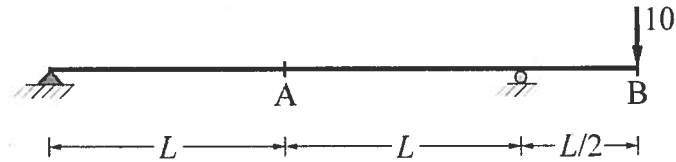


Figure 3: Simply supported girder with overhang under force at the tip

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4. Results involving multiplication or division with a matrix larger than 2×2 will not receive credit.

1. Problem (50% weight)

The continuous beam over two spans of equal length L in Fig. 1 has a flexible middle support with axial stiffness k_s . It is subjected to a vertical force P_v at the middle of the second span. The beam has uniform flexural stiffness EI .

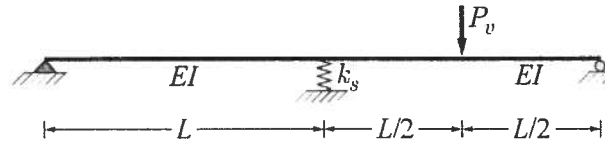


Figure 1: Continuous beam with flexible middle support

You are asked to answer the following questions:

1. What is the value of the spring stiffness k_s in terms of the span length L and the flexural stiffness EI , if the axial force in the spring is 65% of the applied force P_v ?
2. For the spring stiffness value of the preceding question determine the vertical translation at the point of load application.

2. Problem (50% weight)

The structural model in Fig. 2 consists of 2 *inextensible* frame elements a and d with flexural stiffness EI and of 2 *inextensible and inflexible* elements b and c. The structure is subjected to a uniformly distributed load w of 10 units in elements a and b, as Fig. 2 shows.

You are asked to answer the following questions:

1. Determine the vertical translation at node 2 in terms of EI .
2. Draw the bending moment diagram under the given loading.

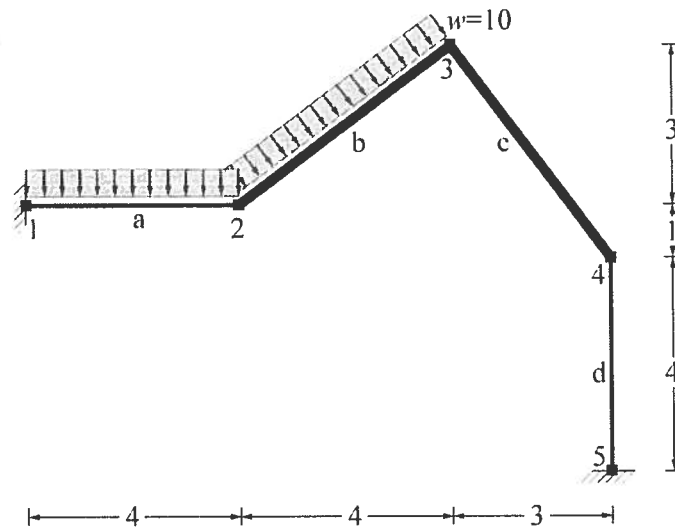


Figure 2

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1. Problem (50% weight)

The structural model in Fig. 1 consists of three *inextensible* frame elements a, b, and c.

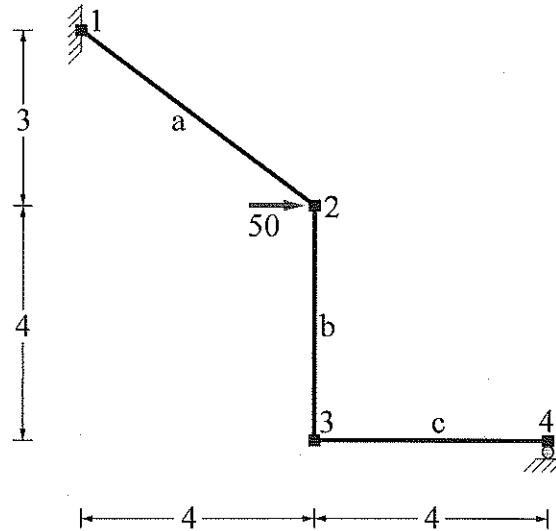


Figure 1: Frame with linear elastic, perfectly plastic basic force-deformation relation

The frame elements exhibit a linear elastic, perfectly plastic basic force-deformation relation with very high plastic axial capacity N_p . The flexural stiffness EI of the frame elements is equal to 40,000 units. The frame element a has plastic flexural capacity M_p of 80 units, while the plastic flexural capacity M_p of the elements b and c is 100 units. *It is assumed that the presence of an axial force in the frame elements does not affect their plastic flexural capacity.*

The structure is subjected to a horizontal nodal force of 50 units at node 2, as Fig. 1 shows.

You are asked to answer the following questions:

1. Determine the load factor λ_1 at the formation of the first plastic hinge.
2. Determine the collapse load factor λ_c .
3. Draw the bending moment distribution $M(x)$ at incipient collapse.

Note: Questions (2) and (3) can be solved independently of the first.

2. Problem (50% weight)

The structural model in Fig. 2 consists of 3 frame elements a, b and c, and two truss elements d and e. The frame elements are *inextensible*. In addition, the elements a and c are *inflexible*. The frame element b has flexural stiffness EI of 10,000 units. The truss element d has axial stiffness EA of 10,000 units, and the truss element e has axial stiffness EA of 5,000 units. The truss element d is prestressed with an initial prestressing force q_0 of -10 units (compressive force).

The structural model is subjected to a uniformly distributed load w of 10 units in element c, as Fig. 2 shows.

You are asked to answer the following questions:

1. Determine the vertical translation at node 2.
2. Determine the basic element forces in all elements.
3. Draw the bending moment diagram under the given loading.

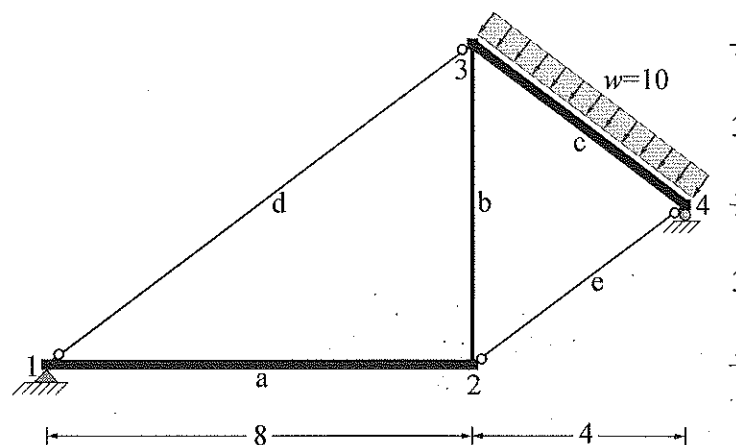


Figure 2