

Preliminary Examination - Dynamics**Problem 1** (30% weight)

An undamped SDOF system with mass m and stiffness k is initially at rest and is then subjected to a full-cycle sine pulse of ground motion, as shown in Figure 1. The natural period of the system is $T_n = t_d/2$. Determine the deformation, $u(t)$, for the time interval $0 < t < t_d$.

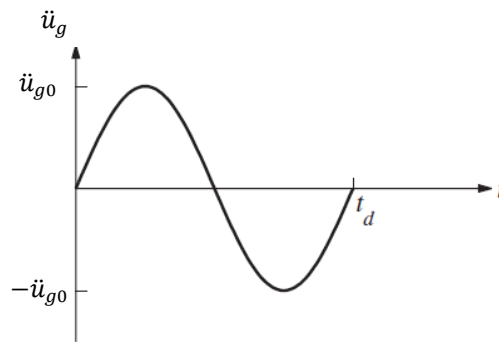


Figure 1

Problem 2 (30% weight)

The beam in Figure 2 has a uniform mass per unit length of $m(x) = m$ and a uniform bending stiffness of $EI(x) = EI$. Using Rayleigh's method, estimate the fundamental natural frequency of the beam.

Recall:
$$\tilde{m} = \int_0^L m(x) [\psi(x)]^2 dx$$

$$\tilde{k} = \int_0^L EI(x) [\psi''(x)]^2 dx$$

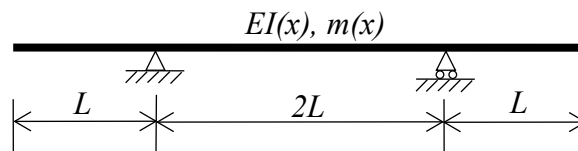


Figure 2

Problem 3 (40% weight)

Figure 3a shows a small mass m and a large mass $5m$ that can only translate in the vertical direction.

- Determine the natural frequencies and mode shapes.
- The system experiences a vertical ground motion which can be described by the response spectra in Figure 3b. Using modal superposition, estimate the maximum force in the spring between the two masses during the earthquake. Assume that all modes of the system have 5% damping.

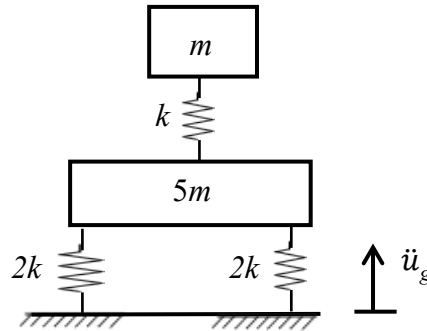
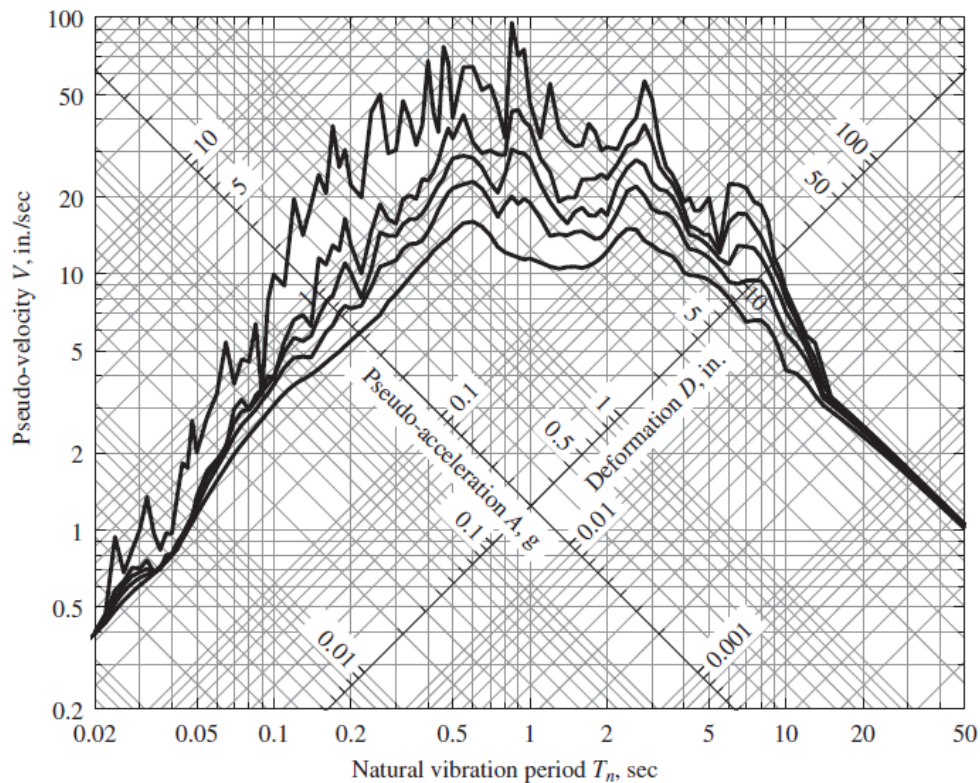


Figure 3a



* Spectra shown for $\zeta = 0, 2, 5, 10, 20\%$

Figure 3b

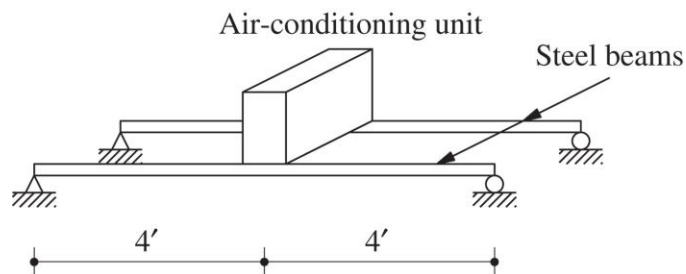
Ph.D. Preliminary Examination: Dynamics

Name: _____

QUESTION 1 [30%]:

An air-conditioning unit weighing 1200 lb is bolted at the middle of two parallel simply supported steel beams ($E = 30,000$ ksi) as shown below. The clear span of the beams is $L = 8$ ft. The second moment of cross-sectional area of each beam is $I = 10$ in⁴. The motor in the unit runs at 300 rpm and produces an unbalanced vertical force of 60 lb at this speed. Neglect the weight of the beams and assume $\zeta = 1\%$ viscous damping in the system. Determine the amplitudes of:

- 1) steady-state deflection, and 2) steady-state acceleration (in g's) of the beams at their midpoints which result from the unbalanced force.

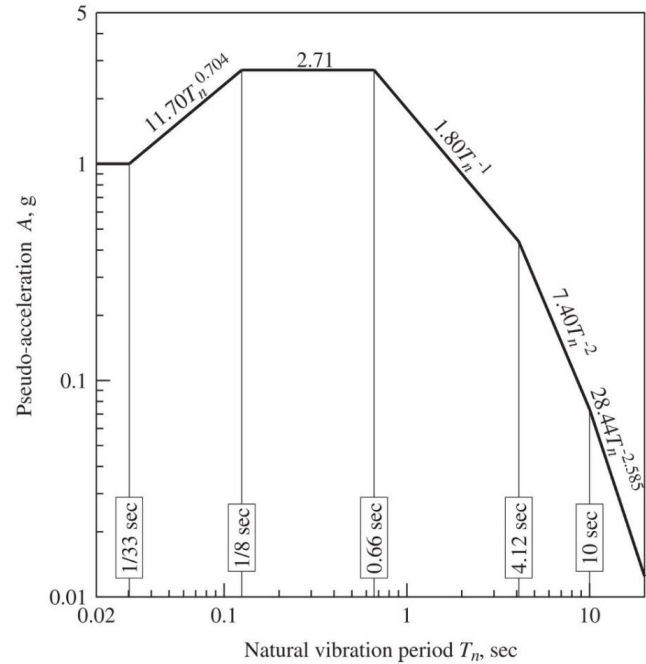
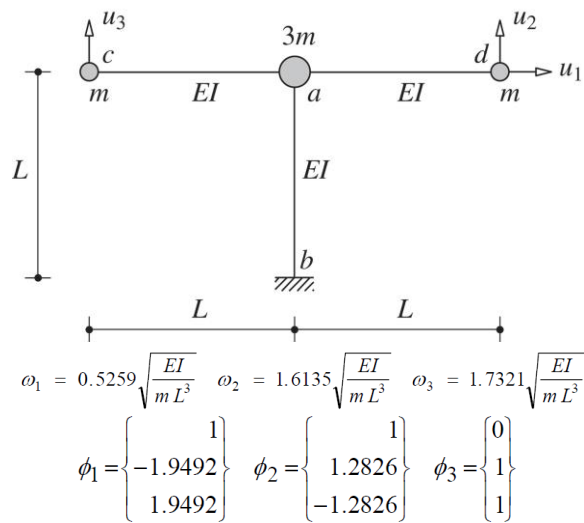


Hints:

$\delta_{\max} = \frac{Pl^3}{48EI}$	Deformation response factor: $R_d = \frac{1}{\sqrt{[1 - (\omega/\omega_n)^2]^2 + [2\zeta\omega/\omega_n]^2}}$	$g = 386$ in/sec²
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QUESTION 2 [70%]:

The umbrella structure shown below has 3 DOF with the listed eigen solution. It is made of standard steel pipe and has the following properties: $I = 28.1 \text{ in}^4$, $E = 29,000 \text{ ksi}$, weight = 18.97 lb/ft , $m = 1.5 \text{ kips/g}$, and $L = 10 \text{ ft}$. It is required to determine the peak response of this structure to horizontal ground motion characterized by the design spectrum shown below (for 5% damping) scaled to $0.20g$ peak ground acceleration and using the SRSS modal combination rule. It is required to perform the following: 1) Check that the distributed weight of the umbrella structure can be neglected (i.e. less than 10%) compared to the lumped weights and find the corresponding mass matrix, 2) Compute the displacements u_1 , u_2 , and u_3 , and 3) Compute the bending moments at the base of the column "b" and at location "a" of the beam.

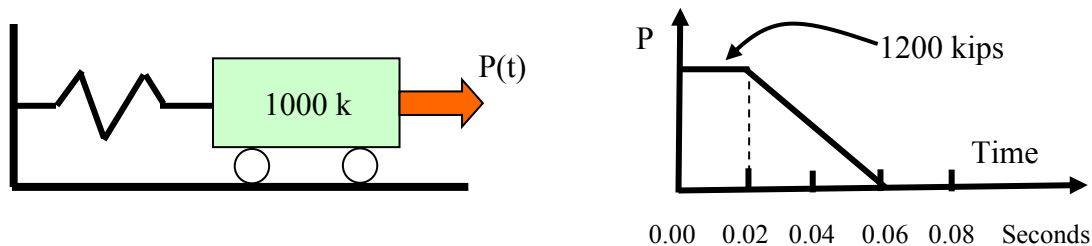


Elastic pseudo-acceleration design spectrum for ground motion: $\ddot{u}_{go} = 1g$ ($g = 386 \text{ in/sec}^2$) and $\zeta = 5\%$.

Preliminary Examination in Dynamics

Problem 1

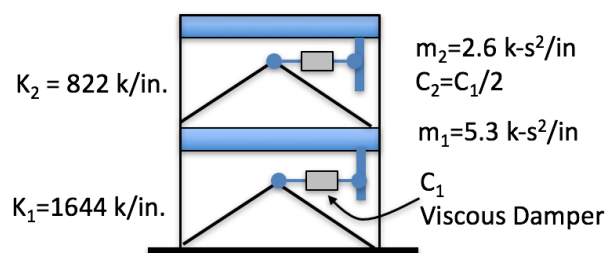
The simple structure shown below weighs 1,000 kips and has a period of 1.25 sec. It has no viscous damping. It is subjected to the impulsive load shown in the figure. If the structure is to be designed such that it has only **one-third** the strength necessary to keep it elastic, what is the expected peak lateral displacement of the structure?



Problem 2

Consider the elastic two degree-of-freedom shear building shown below. Stories and floors are numbered starting at the bottom of the building (like an elevator). Floor masses and story stiffnesses are indicated in the figure below. You should **ONLY** consider the first mode dynamic response to horizontal base seismic excitations. The structure may be assumed to have zero inherent viscous damping. It is desired to increase the effective viscous damping in the first mode to 15% by adding linear fluid viscous dampers between each floor as shown below. The viscous dampers may be assumed to provide **NO** stiffness to the structure, and can be assumed to provide a transverse velocity dependent force between each level of $F_d = C_v$, where C is the damping coefficient, and v is the relative lateral velocity between adjacent floors. Identical dampers are to be placed in each story. The braces used to hold the dampers in place may be assumed to be inextensible. You may make any other common simplifying assumptions in your analyses, but please indicate all such assumptions.

Determine the value of C_1 necessary to achieve the 15% effective viscous damping ratio in the first mode (note $C_2 = C_1/2$).



Ph.D. Preliminary Examination: Dynamics

QUESTION 1 [20%: Part a=15%; Part b=5%]:

- a) The response spectrum ($\zeta = 0.02$) for the El Centro earthquake is shown in **Fig. 1** in terms of pseudo-displacements (D [m]), pseudo-velocities (V [m/sec]), and pseudo-accelerations (A [g]). Calculate the four values with question marks. Note that the acceleration of gravity is to be taken as $g = 9.8 \text{ m/sec}^2$.
- b) From part (c), if a structural engineer underestimated the stiffness of an equivalent SDOF, comment on the safety of the design regarding equipment attached to this SDOF if it is sensitive to: (i) deformation; (ii) acceleration.

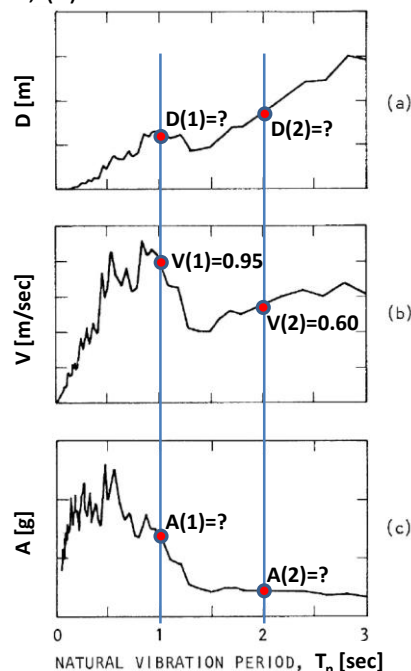


Fig. 1: (a) Displacement, (b) Pseudo-velocity, (c) Pseudo-acceleration response spectra ($\zeta=0.02$) for El Centro ground motion

QUESTION 2 [80%: Part a(i)=25%; Part a(ii)=20%; Part a(iii)=5%; Part b(i)=25%; Part b(ii)=5%]:

- a) Let's take a look at a reactor building at the Fukushima Daichi nuclear power plant, shown in **Fig. 2a**. A two-story frame shown in **Fig. 2b** is used to model the reactor building. Note the distribution of masses (m vs. $8m$) that reflect the weight of the spent fuel pool. The building has the following properties: $m = 5 \text{ kip-sec}^2/\text{in}$, $k = 100 \text{ kips/in}$. The primary containment is considered to be rigid at this preliminary stage of the analysis. There is a gap sized Δ between the primary containment and the spent fuel pool. A pseudo-acceleration design spectrum given in **Fig. 2c** is derived from the 2011 Great Tohoku earthquake records. It is required to conduct the following:
- Determine natural frequencies and mode shapes of the building.

- ii. Using an appropriate modal combination (MC) rule, determine the smallest size of the gap Δ_{req} (rounded to nearest $\frac{1}{4}$ in.) to avoid pounding given the design spectrum.
 - iii. Briefly, justify your choice of the MC rule (no calculations needed)
- b) A tsunami wave force idealized, as shown in **Fig. 2d** with two equivalent lateral forces, $P(t)$ at the spent fuel pool level and $P(t)/2$ at the roof level, where $P(t)$ is modeled using rectangular pulse function (duration $t_d=1.0$ sec., **Fig. 2e**). Conduct the following:
- i. Determine the maximum lateral displacement of the reactor building at the spent fuel pool level. [Hint: The dynamic response factor $R_d=2.0$ if $t_d/T_n \geq 0.5$]
 - ii. Assuming the primary containment is unaffected by the tsunami wave, is the gap size Δ_{req} determined considering earthquake load in (a(ii)) sufficient to prevent pounding due to tsunami wave load? State clearly Yes or No with justification.

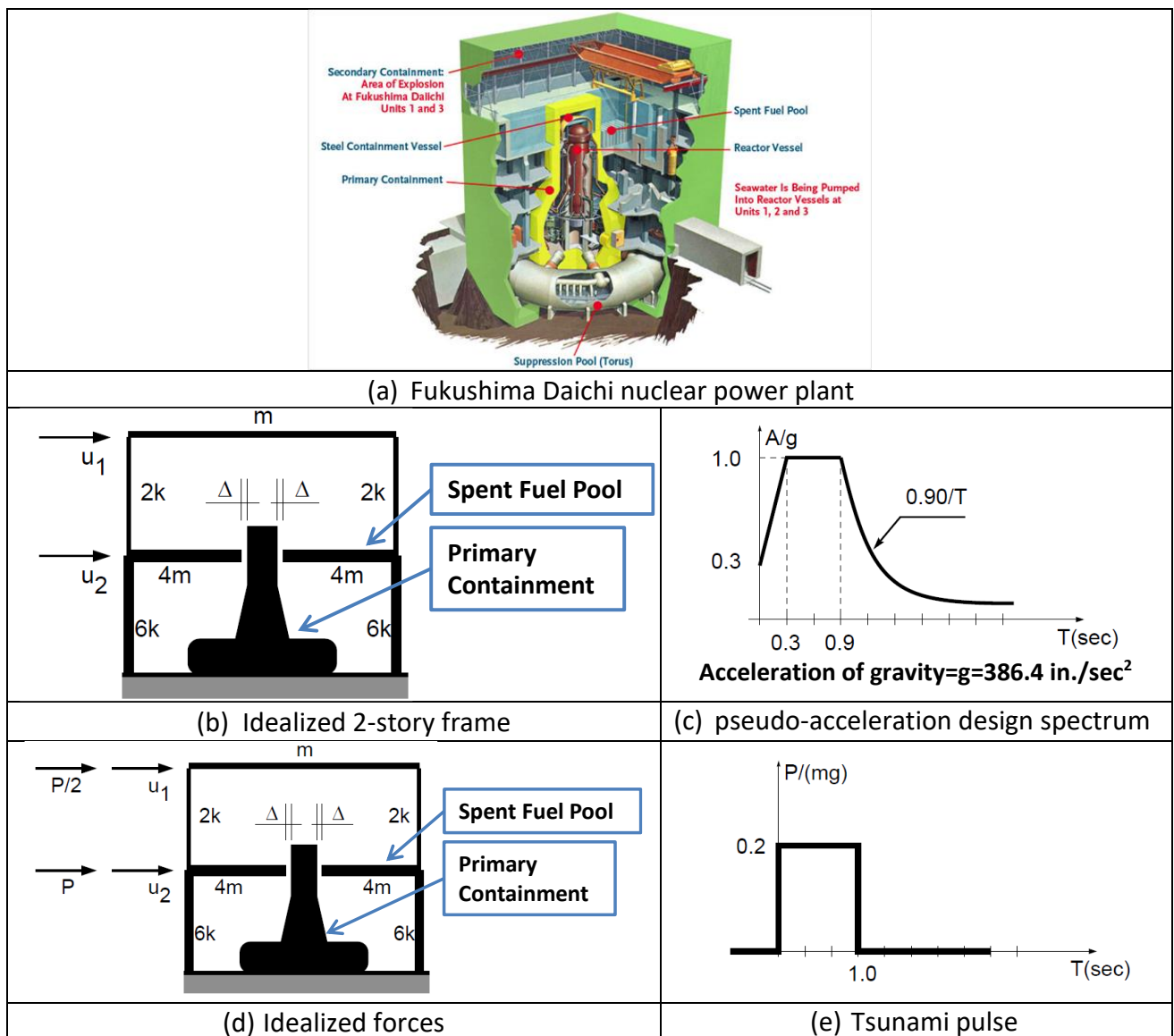
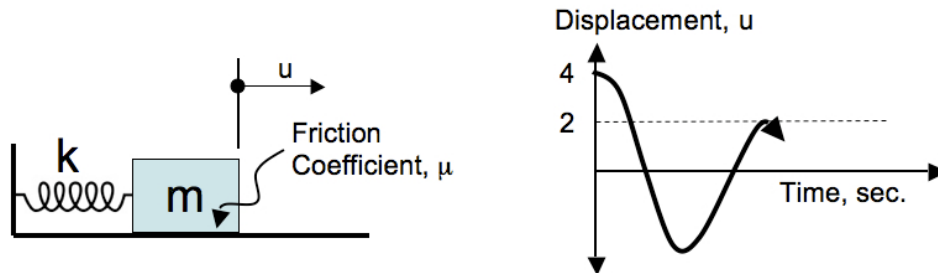


Fig. 2: Problem 2 details

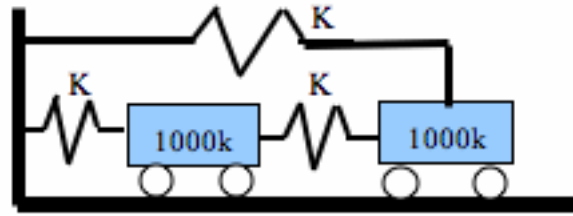
Preliminary Examination in Dynamics

1. Consider a simple single degree-of-freedom system shown below. The rigid block with mass **m** slides on the horizontal surface. The block is attached to a rigid support by a linear elastic spring with stiffness **k**. Movement of the block is also resisted by friction between the block and the sliding surface. The horizontal force needed to overcome the friction equals $F_f = \mu W$, where μ is an unknown friction coefficient and W is the weight of the mass block. W equals 200 kips and **k** equals 133 kips/inch. The mass is moved to the right 4 inches and then released. What friction coefficient is needed so that after one cycle of oscillation, the amplitude of horizontal oscillation is reduced to 2 inches? (40 points)



2. Consider the two-degree of freedom system shown below. The structure is assumed to have no inherent viscous or hysteretic damping. All of the springs shown have the same stiffness, but the value of stiffness is not specified. However, the first mode period of the structure is given as 2 seconds. The structure is subjected to a horizontal earthquake excitation represented by a simplified response spectrum $D_n=3T$, where the spectral displacement D_n is measured in inches, and period is specified in seconds. Modal contributions to response can be estimated using SRSS methods. It may be assumed that peak responses in each mode can be represented by the specified spectrum. (60 points)
- What is the stiffness required of the springs (all equal)?
 - What is the expected maximum force in the spring located between the two masses?
 - If linear viscous dampers are placed in parallel with all of the springs shown, and they each exhibit the relation $F_d = C\dot{v}$, where $\dot{v}$ represents the relative velocity acting across the damper, what is the value of C (expressed in k-sec/in) to give an effective viscous damping ratio of 15% in the first mode of vibration?

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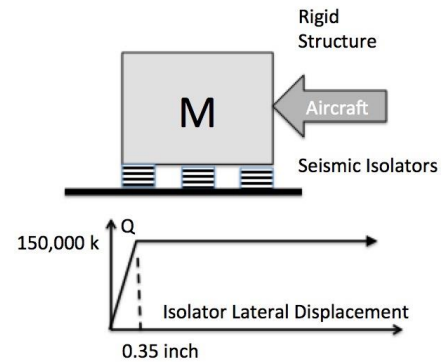


Preliminary Examination in Dynamics

1. Consider the seismically isolated structure to the right. Using engineering fundamentals, estimate the maximum lateral displacement of the isolators, if an aircraft strikes the structure.

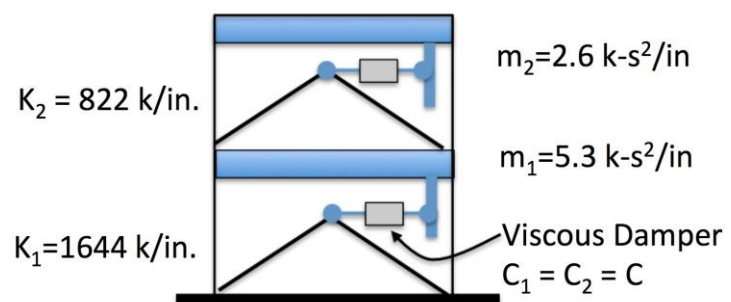
The structure above the isolators weighs 1,000,000 kips and can be considered completely rigid. The isolators have elastic-perfectly plastic properties in the lateral direction as shown to the right. Vertically, the isolators are inextensible. The aircraft weighs 450 kips and is traveling horizontally at 800 ft/sec at the time of impact.

The aircraft “sticks” to the structure during/following the impact. The duration of the impact loading is 0.1 sec, which may be considered significantly shorter than the natural period of the structure. Energy dissipation of the structure, other than in the isolators, should be considered minimal (zero).



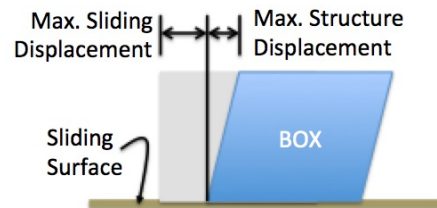
2. Consider the elastic two degree-of-freedom shear building shown below. Floor masses and story stiffnesses are indicated in the figure below. You should ONLY consider the first mode dynamic response to horizontal base seismic excitations. The structure may be assumed to have zero inherent viscous damping. It is desired to increase the effective viscous damping in the first mode to 10% by adding linear fluid viscous dampers between each floor as shown below. The viscous dampers may be assumed to provide NO stiffness to the structure, and can be assumed to provide a transverse velocity dependent force between each level of $F_d = C\dot{v}$, where C is the damping coefficient, and $\dot{v}$ is the relative lateral velocity between adjacent floors. Identical dampers are to be placed in each story. The braces used to hold the dampers in place may be assumed to be inextensible. You may make any other common simplifying assumptions in your analyses, but please indicate all such assumptions.

Determine the value of C necessary to achieve the 10% effective viscous damping ratio in the first mode (note C is to be the same for both stories).



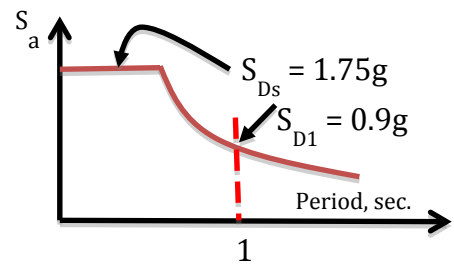
Preliminary Examination in Dynamics

- Consider the simple single degree-of-freedom system shown on the right. The box-like structure slides sideways on the horizontal surface. The box is NOT rigid, but deforms in simple shear. It is strong enough that it will remain elastic during the seismic response, but it may slide. The box's height to width aspect ratio is such that it will slide before it is able to uplift. Sliding of the block is resisted by friction between the block and the sliding surface. The horizontal force needed to overcome the friction equals $F_f = \mu W$, where μ is the friction coefficient and W is the weight of the block. Here, $\mu = 0.5$ and W equals 20 kips. The box has a fundamental period of 1.2 seconds.



The structure will be subjected to horizontal ground shaking corresponding to the response spectrum shown. You will need to make some reasonable simplifying assumptions. Be sure to describe them.

Estimate the maximum displacement of what occurs over the height of the structure, and the expected amount of sliding of the structure.



- Consider the two-degree of freedom system shown below. It is subjected to a horizontal earthquake excitation represented by a simplified response spectrum $D_n = 4T$, where the spectral displacement D_n is measured in inches, and the period is specified in seconds. It is desired that the structure be proportioned so that the expected peak horizontal response of the mass on the right is 8 inches. Modal contributions to response can be estimated using SRSS methods. It may be assumed that peak responses in each mode can be represented by the specified spectrum.

- What stiffness parameter K is required for the springs to achieve the stipulated peak displacement?
- What is the expected maximum force in the spring located between the two masses if the spring stiffness K computed in part a is used?

