On Seismic Response of Stiff and Flexible Retaining Structures

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ABSTRACT

This paper presents an overview of the results of experimental and analytical studies of the seismic response of stiff and flexible retaining structures. These studies were motivated by the fact that the current seismic design methodologies based on the work of Mononobe-Okabe in the 1920’s to predict very large dynamic forces in areas of high seismicity. Yet, there is no evidence of systematic failures of retaining structures in major earthquakes even when the ground accelerations clearly exceeded the design assumptions. The experimental program consisted of a series of geotechnical centrifuge model studies with different types of structures with cohesionless and cohesive backfill. Overall, the results of these studies show that the Mononobe-Okabe method of analysis provides a reasonable upper bound for the response of stiff retaining structures and that flexible retaining structures experience loads significantly smaller than those predicted by this method. Moreover, for deep embedded structures the dynamic forces do not continue to increase with depth and gradually become a small fraction of the overall load on the walls.

Introduction

The introduction of more stringent seismic design provisions in recent updates of design codes, e.g. IBC 2012 and FEMA 750, has increased the demand on seismic design of retaining walls and basement structures and, hence, there is a need for appropriate analysis and design methodology. While not all codes are prescriptive in specifying a particular methodology, the most commonly recommended analyses for cantilever and gravity structures are based on a limit equilibrium method developed in the 1920’s in Japan by Okabe (1924) and Mononobe and Matsuo (1929), generally referred to as the Mononobe-Okabe method. While various modifications of this method have been introduced since, e.g. Seed and Whitman (1970) and Mylonakis et al. (2007), the principal problem for a designer is that at high accelerations, > 0.5 g, these methods predict very large dynamic forces, which appear unrealistic in view of actual experience in recent earthquakes. The problem of predicting high seismic loads becomes even more pronounced in the design of “non-yielding walls”, defined as structures based on rock or very stiff soil that will not deflect more than 0.002H (FEMA 750), which is based on a solution by Wood (1973). An additional complication is that the recommended design procedures make very little difference between different types of retaining structures. Figure 1 is a schematic representation of some of the typical settings and configurations of retaining structures. Clearly, free-standing cantilever structures that can translate and rotate will respond differently than cantilever sides of channel structures that can flex, but cannot translate, and structures on sloping ground present yet another, completely different scenario. Moreover, structures retaining native ground will experience different loading than structures retaining cohesionless or cohesive backfill.

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Sitar et al. (2012) present a detailed review of the different methods of analysis and their underlying assumptions and, therefore, the focus of this paper is first on pointing out the fundamental differences in the various approaches that lead to the reported results. Recent experimental results in addition to observations from recent earthquakes are presented to show that the above mentioned traditional analysis methods do not adequately represent the actual seismic demand and that they are indeed conservative.

**Observed Response in Recent Earthquakes**

Before considering the theoretical aspects of the methods of analysis, it is worthwhile to note the observed performance of retaining structures in recent major earthquakes. A review of the performance of basement walls by Lew et al. (2010), drawing on experience from the 1989 Loma Prieta, 1994 Northridge and 1995 Kobe earthquakes, concluded that failures of basement walls or deep excavation bracing in earthquakes were rare. In this context it is important to note that many of the older structures were either not at all designed for seismic loading, or even if they were the designed for seismic loading, the design ground motions were significantly smaller than those actually experienced by the structures. When failures did occur they were either liquefaction-triggered failures of waterfront structures retaining saturated backfill, or structures on slopes and retaining sloping backfill (Sitar et al. 2012). Most recently, no significant damage or failures of retaining structures occurred in the 2008 Wenchuan earthquake in China, or in the great subduction earthquakes in 2010 in Chile (Verdugo et al. 2012) or in 2011 in Japan (Sitar et al. 2012). Figure 2 shows a conventional retaining wall supporting a cut slope adjacent to a fault trace in Sichuan 2008. Except for the offset along the fault (Figure 2a), there were no other signs of distress. On the other hand, structures with design flaws have experienced distress. Figure 3 shows a cantilever wall, which rotated about its base in the 2014 Iquique earthquake in Chile. The wall does not have a footing and has essentially no embedment. However, it is not clear if the rotation was caused by the inertia of the wall itself or due to the pressure of the backfill. An adjacent similar height wall showed no distress. Most importantly, there is no evidence of a systemic problem with well executed, conventional, static or minimal seismic retaining wall design even under quite severe loading conditions. For example Clough and Fragaszy (1977) found that reinforced concrete cantilever structures, well designed and detailed for static loading, performed without any sign of distress at accelerations up to about 0.4 g. A similar conclusion was reached by Seed and Whitman (1970), who suggested that conventionally designed gravity structures should perform satisfactorily under seismic loading up to 0.3 g.
Figure 2. Conventional retaining structures in the epicentral region of the Wenchuan 2008 Earthquake. Note the fault break in the picture on the right.

Figure 3. Reinforced concrete wall without a footing rotated during the 2014 Iquique, Chile, earthquake. Courtesy of G. Candia
Methods of Analysis and Design

Conventional Gravity and Cantilever Walls ("Yielding Walls")

Cohesionless Backfill

Conventional gravity and cantilever walls, i.e. walls that can deflect, rotate, and translate, are most commonly designed using the Mononobe-Okabe (M-O) or, particularly in the US, using the Seed and Whitman (1970) simplified approach. Thus, it is of interest to review both methods and to point out the most significant differences. Implicit in both methods is the development of a Coulomb wedge as shown for the M-O method in Figure 4a. Hence, an inherent assumption is that the retained soil mass behaves as a rigid body and there is no phase difference between the response of the structure and the soil. Most importantly, the application of the combined static and dynamic force is at 1/3H and the active thrust is given by:

\[ P_{AE} = 0.5 \gamma H^2 (1-k_v) K_{AE} \]  

Where, 

\[ K_{AE} = \frac{\cos^2(\phi - \theta - \beta)}{\cos \theta \cdot \cos^2 \beta \cdot \cos(\delta + \beta + \theta) \cdot \left[ 1 + \frac{\sin(\phi + \delta) \cdot \sin(\phi - \theta - i)}{\cos(\delta + \beta + \theta) \cdot \cos(\phi - i)} \right]^2} \]  

where \( \gamma \) = unit weight of the soil, \( H \) = height of the wall, \( \phi \) = angle of internal friction of the soil, \( \delta \) = angle of wall friction, \( \beta \) = slope of the wall relative to the vertical, \( \theta = \tan^{-1}\left(k_h/(1-k_v)\right) \), \( k_h \) = horizontal acceleration (in g), and \( k_v \) = vertical acceleration (in g).

A major limitation of equation (2) is that it increases exponentially and does not converge if \( \theta < \phi - \beta \) (e.g. Kramer, 1996), which for typical values of angle of internal friction means accelerations in excess of 0.7 g. The alternative introduced by Seed and Whitman (1970) is to separate the total force on the wall into its static and dynamic components and the simplified expression of the active thrust is given by:

\[ \Delta P_{AE} = \frac{1}{2} \gamma H^2 \Delta K_{AE} \quad \text{and} \quad \Delta K_{AE} \Delta P_{AE} = \frac{1}{2} \cdot \gamma \cdot H^2 \cdot \Delta K_{AE} \sim \frac{3}{4} k_h \]

where \( k_h \) is the horizontal ground acceleration as a fraction of the acceleration of gravity. This approximation is asymptotically tangent to the M-O solution at accelerations below about 0.4 g and it remains linear throughout. The most significant difference, however, is in the point of application of the resultant of the dynamic force increment, which Seed and Whitman (1970) place at 0.6H (Figure 4b), which has led to the concept of an "inverted triangle" to represent the distribution of the dynamic soil pressure on the retaining structure.

The most significant consequence of moving the point of application of the dynamic force increment to 0.6H is that it essentially doubles the dynamic moment on the structure. While many researchers have weighed in on the issue over the years, it is of note to revisit observations
by Mononobe and Matsuo (1932). Specifically, based on their shaking table experiments with sandy backfill, they note that for a very stiff wall the point of application of the dynamic pressure is 0.622H. However, they also state that for more elastic structure response the point of application should be “$h_c/H = 1/3$ or less” and that is what they use in their analyses. In general, experiments on stiff walls with cohesionless backfill on a rigid base on 1-g shaking tables show the same effect, i.e. the maximum dynamic earth pressure increment occurs at 0.4-0.6H (see e.g. Sherif et al. 1982). In addition to noting that the flexibility of the structure affects the observed dynamic stress increments, Mononobe and Matsuo (1932) also observed that higher degree of compaction also decreased the dynamic soil pressure.

**Cohesive Backfill**

It is important to note that the original M-O method and its derivative’s were developed considering purely cohesionless backfill. However, in many typical situations backfill typically has some amount of cohesion and, therefore, it is important to consider it formally in the analyses. Equation (4) is the full version of Okabe’s (1924) general equation (simplified in Equation 2) that includes a cohesion term as follows:

$$K_{ae} = \frac{\sin(\alpha - \phi + \theta) \cos(\alpha - \beta) \cos(\beta + \delta) + \frac{2c}{\gamma H (1 - k_h)} \cos\beta}{\cos^2 \beta \cos \theta \sin(\alpha - i) \cos(\alpha - \beta - \phi - \delta)}$$

This solution suffers from some of the same limitations as equation (2) in that it becomes indefinite when $k_h > \tan \phi + 2c/\gamma H$. More recently, methods developed by Chen and Liu (1990) and Richard and Shi (1994), for example, do not suffer from the same limitations. The influence of cohesion on the computed seismic coefficient is quite significant and should not be neglected as illustrated in Figure 5. Specifically, Anderson et al. (2008) conclude that the “reduction for typical design situations could be on the order of about 50 percent to 75 percent”. The good observed seismic performance of retaining structures may in be in part due to the presence of cohesion in typical backfills and in native ground.

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**Figure 4. Force equilibrium diagrams for:**

a) M-O method (Mononobe and Matsuo 1929) and

b) Seed and Whitman (1970)
As already mentioned Mononobe and Matsuo (1932) observed that stiffer structures, rigidly attached at the base experience higher seismic loads by granular backfill. This problem was first addressed analytically by Wood (1973) who modeled linearly elastic soil in a container with rigid walls and a rigid base as shown in Figure 6a. As can be seen, the computed dynamic stress increment is zero at the base and maximum at the top of the backfill with the recommended point of application of the resulting force at 0.6H. This approach has been adopted by other researchers and similarly high seismically induced earth pressures were computed, e.g. Matsuo and Ohara (1960), Prakash (1981), Sherif et al. (1982), Ostadan and White (1998) and Ostadan (2005). However, very few structures are perfectly rigid and FEMA 750 (Anderson et al. 2008) explicitly points out that this solution applies to non-yielding walls “founded on rock or very stiff soil”. A non-yielding wall is defined as one with deformations < 0.002H. This same observation was made earlier by Mononobe and Matsuo (1932). Moreover, the solution is strictly applicable only to cohesionless backfill. Thus, care should be exercised in considering this approach in the design of typical retaining structures as it may lead to extremely conservative results.

“Non-yielding”, Stiff Retaining Structures

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Linear Elastic Analytical Solutions

Linear elastic closed form or iterative solutions have been proposed by a number of investigators including Veletsos and Younan (1997), Younan and Veletsos (2000), Zeng (1990), and Steedman and Zeng (1990). While computationally more demanding, these methods offer an alternative to the limit equilibrium methods and have the advantage that they can consider elastic wave propagation, including wave attenuation and the influence of the relative stiffness of the soil and the structure. Although, these methods cannot handle material non-linearity and cannot account for energy dissipation should a gap open between the structure and the retained soil, they provide a rigorous solutions that can be tested against experimental results. When considering realistic wall stiffness/flexibility these methods predict substantially lower pressures than those predicted for rigid walls (Figure 7a). Moreover, Younan and Veletsos (2000) show that the pressure distribution becomes roughly triangular with depth and the point of force application decreases from 0.6 H for a rigid wall to less than 0.3 H for a flexible cantilever wall (Figure 7b). Increased foundation flexibility, i.e. increased rotation flexibility, has a similar effect (Veletsos and Younan, 1997). These results provide theoretical support for the empirical observations of Mononobe and Matsuo (1932) and they are consistent with the results of recent experimental studies presented next.

Experimental Data

While the field observations following earthquakes are very valuable, one of the main limitations is that most commonly information on the actual design and construction is lacking. Hence, except in rare cases, e.g. Clough and Fragaszy (1977), it is not possible to perform a rigorous back analysis of the observed performance. Therefore, experimental results are essential in order to be able to evaluate the validity of the various assumptions and the applicability of the various methods of analysis. To this end the authors have been involved in an extensive program of centrifuge model experiments on different types of structures in both cohesionless and cohesive soils (Al Atik and Sitar, 2010; Sitar et al., 2012; Gerali Mikola and Sitar, 2013; Candia and Sitar, 2013). The centrifuge was chosen for the experimental program because it allows for...
consistent scaling of the critical parameters and the experiments are relatively economical in terms of time and cost. Most importantly, the scale of the models allows for the structures to be founded on soil (Figure 8) and, consequently, avoid the rigid base foundation issue already discussed. Also, as seen in the figure, the centrifuge provides an excellent environment for data collection and the models can be extensively instrumented with strain gauges, accelerometers, pressure sensors, etc. Overall, the experimental program involved a series of experiments with flexible and stiff U-shaped and cantilever structures retaining cohesionless and cohesive soils. All of these structures were 6.5 m high in prototype scale, representing typical height of walls used for highway structures. A separate set of experiments involved stiff, braced structures designed to mimic basement type walls in cohesionless soil. For brevity the specifics of scaling and experimental procedures are omitted herein, as they are described in detail in the publications referred to above. Instead the emphasis is on presentation of the results and their interpretation.

**Dynamic Earth Pressures on Stiff and Flexible Walls**

The two types of structures shown in Figure 8 are both cantilevers with the distinction that in the U-shaped channel configuration the wall is free to deflect but cannot translate or rotate about its base, whereas the free-standing cantilever wall can translate and rotate. While this distinction is important when it comes to the magnitude of the observed dynamic forces it did not seem to affect the distribution of the dynamic stress increment which was roughly triangular, increasing downward, as illustrated in Figure 9. The corresponding computed M-O and Seed and Whitman (S-W) dynamic pressure increments are plotted for reference. This data shows that the point of application of the dynamic load increment on cantilever walls on flexible foundation is roughly at 1/3 H as postulated by Mononobe and Matsuo (1932) and computed analytically by Younan and Veletsos (2000, Figure 7b). Similar observations were made in centrifuge experiments on cantilever walls by Ortiz et al. (1983) and Stadler (1996), and gravity walls by Nakamura (2006). Also, as can be seen, the point of application of the dynamic force at 0.6H suggested by Seed and Whitman procedure (S-W, “inverted triangle”) is not supported by the data.
Figure 9. Dynamic earth pressure distributions directly measured and interpreted from the pressure sensors and strain gage and load cell data and estimated M-O and S-W values (PGAff=0.61, Geraili Mikola and Sitar 2013).

Figure 10. Seismic earth pressure coefficients from centrifuge model tests for stiff walls in cohesionless and cohesive soil (top) and corresponding results for cantilever walls (From Candia and Sitar, 2013).
A more direct way of evaluating the previously discussed conventional analysis procedures is to view the results in terms of the seismic coefficient $\Delta K_{se}$. Figure 10 is a summary of data obtained from centrifuge experiments with cohesionless soil by Geraili Mikola and Sitar (2013) and compacted silty clay by Candia and Sitar (2013). The results are plotted against the peak acceleration in free field at the point of maximum moment on the structure as that seemed to be the simplest measure that can be readily obtained in a typical analysis. The results show that the M-O solution and the S-W approximation provide a reasonable upper bound for stiff structures in cohesionless soil and are about 1/3 of what would have been obtained using the Wood (1973) solution for rigid structures (Candia and Sitar, 2013). Cohesion has a small, but measurable effect, and as the plot shows the experiments with compacted silty clay backfill resulted in values at or below the mean of the data set. As would be expected, the observed seismic coefficient for cantilever walls is significantly lower than what would be predicted by any of the conventional design methods, although there is no difference between cohesive and cohesionless backfill, which may be an artifact of the experimental procedure. Seed and Whitman (1970) suggested that well designed gravity retaining structures should perform well at acceleration up to 0.3g without having been designed for seismic loading. The data for cantilever structures presented above supports their assertion, as the observed seismic coefficient are very small for accelerations below 0.3g, as previously suggested by Al Atik and Sitar (2010) and Sitar et al. (2012).

**Dynamic Earth Pressures on Deep Stiff Walls**

The results discussed above are applicable to relatively shallow structures up to about 7 m in height/depth and it may be reasonable to extrapolate these results to taller free-standing retaining structures in level ground. However, the same is not possible for more deeply embedded walls, such as deep basement walls, since the seismic earth pressure increment is bound to decrease with depth. This topic is currently an active area of our research and a set of centrifuge experiments was performed on a very stiff braced deep wall, 13.3 m deep and founded on 5.5 m of medium dense sand in prototype dimensions. The structure consisted of two thick walls with three levels of stiff cross braces, as shown in Figure 11. The bracing was instrumented with load cells in order to obtain a direct measurement of loads because the readily available earth pressure sensors, while providing satisfactory relative values, do not provide reliable absolute values. Consequently, the structure was very stiff, albeit not completely rigid. Other instrumentation included accelerometers and LVDT’s to measure site response and to measure transient and permanent deformations and all tests were performed at 36g. In contrast to the experiment layout for the shallow embedded structures (Figure 8), the structure was placed in the center of the container in order to minimize any potential boundary effects.

Figure 12 shows the lateral earth pressures observed in two shaking events in the centrifuge. As can be seen, the maximum dynamic earth pressure increment is equivalent to $K_0$ in the upper part of the deposit, reaching a maximum of about 0.1 $\gamma H$ at about 0.3H. It then decreases with depth. The principal difference between the two events is the number of cycles during which the earth pressure approaches the maximum is much greater for the Kobe-Takatori record than for Loma Prieta record, while the envelopes of maximum dynamic earth pressure are similar. Although, we present these results in dimensionless form, it is not certain that these results can be immediately scaled to structures with different embedment depths or to structures that have
slab or strut at the top of the wall. This is currently an active area of our continued work. However, this data shows that the application of the Wood (1973) solution or any other type of simple monotonically increasing or decreasing stress increment with depth is not warranted for deep stiff structures and basement walls. Moreover, these results suggest that the problem does not lend itself to simple analytical solution and it will be further explored using numerical methods.

Figure 11. Layout of the centrifuge model of a deep stiff structure.

Figure 12. Dynamic earth pressure increment distribution for two different events as a function of depth from centrifuge experiments on a deeply embedded structure.
Conclusions

A review of traditional methods of analysis shows that the flexibility of the retaining structure and its foundation plays a very important role in the magnitude of the predicted seismic loads and that increased wall flexibility significantly decreases their magnitude. This has been first noted by Mononobe and Matsuo (1932) and since then shown to be the case analytically by Younan and Veletsos (2000) among others. The results of an extensive set of centrifuge experiments modeling retaining structures with cohesionless and cohesive backfill and observations of seismic performance of conventional retaining structures provide further support for these conclusions. Moreover the line of action of the resulting forces is shown to be roughly at 1/3 H for typical cantilever or gravity retaining structures of moderate height.

The experimental results also show that the traditionally used Mononobe-Okabe and Seed and Whitman methods of analysis provide a reasonable upper bound for predicted seismic loads on retaining structures. On the other hand, there is no evidence to support the further use of the Wood (1973) solution and its derivatives except in special cases of very stiff structures on very stiff or rock foundation with relatively loose backfill.

Finally the results of recently completed centrifuge experiments on a model of a deeply embedded, stiff, retaining structure show that seismic earth pressures increase only moderately with depth and are a small fraction of the static pressure at depth. Further experimental and analytical work is needed and is in progress to further elucidate this particular case. Finally, due to the complexity of the various types of retaining structures, ultimately, well instrumented and documented case histories are needed to fully assess the range of potential problems and their solution.

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